6.1 – 6.3 Review

1) Find the component form, magnitude and direction of $\overrightarrow{PQ}$.
Given that P = (-7, 8) and Q = (-9, 2)

Terminal − Initial

$\langle -9+7, 2-8 \rangle$

$\langle -2, -6 \rangle$

$mag = \sqrt{(-2)^2+(-6)^2}$

$= \sqrt{40}$

$\tan^{-1}\left(\frac{-6}{-2}\right) = 71.57°$

$180 + 71.57 = 251.57$

2) Find the component form of the indicated vector $2\mathbf{u} - 3\mathbf{v}$.
$\mathbf{u} = \langle -2, 3 \rangle$ $\mathbf{v} = \langle -1, -1 \rangle$

$2\langle -2, 3 \rangle - 3\langle -1, -1 \rangle$

$\langle -4, 6 \rangle - \langle -3, -3 \rangle$

$\langle -4-(-3), 6-(-3) \rangle$

$\langle -1, 9 \rangle$

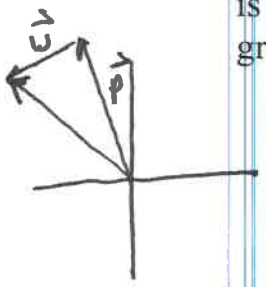
3) Find the unit vector in the direction of the given vector. Write your answer in component form. Let $\mathbf{u} = \langle 6, 9 \rangle$.

$mag = \sqrt{6^2+9^2}$

$= \sqrt{117}$

$\left\langle \frac{6}{\sqrt{117}}, \frac{9}{\sqrt{117}} \right\rangle$

4) A plane is flies on a compass heading of 100 degrees at 275 mph. The wind is blowing at an angle of 205 degrees at 30 mph. What is the true course and ground speed of the plane?



$\vec{p} = \langle 275\cos 100°, 275\sin 100° \rangle$ $\vec{w} = \langle 30\cos 205, 30\sin 205 \rangle$

$\vec{p} + \vec{w} = \langle 275\cos 100 + 30\cos 205, 275\sin 100 + 30\sin 205 \rangle$

$\langle -74.94, 285.84 \rangle$

$Speed = \sqrt{(-74.97)^2+(285.14)^2}$

$= 268.80 \text{ mph}$

$\tan^{-1}\left(\frac{258.14}{-74.97}\right) = -73.81$

2nd Quadrant

$-73.81° + 180 = 106.19°$

5) Find $\mathbf{a} \cdot \mathbf{b}$, given $\mathbf{a} = \langle 2,5 \rangle$, *and* $\mathbf{b} = \langle -3,8 \rangle$

Dot Product

$(2)(-3) + (5)(8)$

$-6 + 40$

34

6) Find the angle between the two vectors.

$\mathbf{u} = \langle 2, -3 \rangle$, $\mathbf{v} = \langle -4, 5 \rangle$

$|u| = \sqrt{2^2+(-3)^2}$

$= \sqrt{13}$

$|v| = \sqrt{(-4)^2+5^2}$

$= \sqrt{41}$

$\cos\theta = \frac{u \cdot v}{|u||v|}$

$\frac{(2)(-4) + (-3)(5)}{(\sqrt{13})(\sqrt{41})}$

$\frac{-23}{\sqrt{533}}$

$\cos^{-1}\left(\frac{-23}{\sqrt{533}}\right) = 175.03°$

7) Determine if the vectors are parallel or orthogonal. If the vectors are orthogonal, prove using the dot product.

Start with Dot product

A) $\mathbf{u} = \langle 5,4 \rangle$, $\mathbf{v} = \langle 4,-5 \rangle$

$(5)(4) + (4)(-5)$

$20 + (-20)$

0

orthogonal

B) $\mathbf{u} = \langle -3,2 \rangle$, $\mathbf{v} = \langle 3,-2 \rangle$

$\vec{u} = \frac{2}{-3}$ $\vec{v} = \frac{-2}{3}$

$(-3)(3) + (2)(-2)$

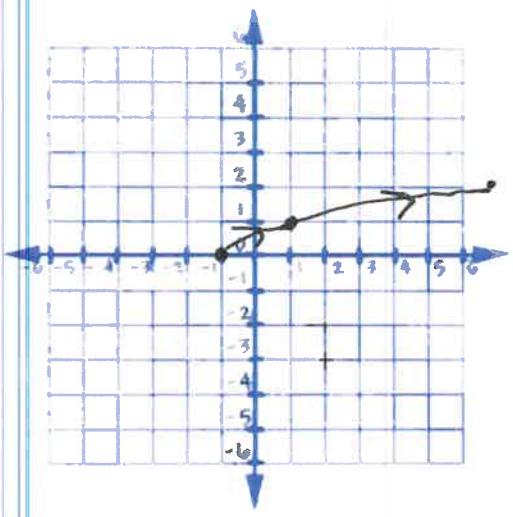
$-9 + (-4)$

-13

Parallel

8) a) For the given parameter interval, graph the parametric equations.
b) Then, eliminate the parameter and identify the graph of the parametric curve

$x = 2t^2 - 1$ $y = t$ on the interval $0 \le t \le 2$



t	x	y	
0	-1	0	(-1, 0)
1	1	1	(1, 1)
2	7	2	(7, 2)

$x = 2t^2 - 1$

$x + 1 = 2t^2$

$\frac{x+1}{2} = t^2$

$t = \pm\sqrt{\frac{x+1}{2}}$

$y = t$

$y = \pm\sqrt{\frac{x+1}{2}}$

Square Root Function

Find a parametrizations for the curve.

9) The line through the points A = (-1, 5) and B = (2, 7)

$t = 0$

$x = -1 + \Delta t$

$-1 + (2 - (-1))t$

$x = -1 + 3t$

$y = 5 + \Delta t$

$y = 5 + (7 - 5)t$

$y = 5 + 2t$

$-\infty < t < \infty$

10) The men's horseshoe pitching court has metal stakes **40 feet apart**. The stakes stand **18 inches** out of the ground.

a. Alan pitches a horseshoe at **48 feet per second**, at a **12° angle** to the ground. He releases the horseshoe at about **3 feet above** the ground and **2 feet in front** of the stake at one end. **Write parametric equations** modeling a typical throw.

$x = 48t\cos 12 + 2$

$y = -16t^2 + 48t\sin 12 + 3$

b. How long is the thrown horseshoe in the air?

$0 = -16t^2 + 48t\sin 12° + 3$

$t = \dfrac{9.98 \pm \sqrt{9.98^2 - 4(-16)(3)}}{-32}$

$t = -.22$ or $t = .8455$

c. How close to 40ft is the horizontal component when the horseshoe hits the ground?

$x = 48(.8455)\cos 12° + 2$

$= 41.657$

1.657 ft past stake

11) Determine whether a baseball hit from a height at 1.5 feet, at a speed of 135 feet per second, and at an angle of 32° relative to level ground will clear a 10-foot wall 425 feet away.

$x = 135t\cos 32°$

$425 = 135t\cos 32$

$t = \dfrac{425}{135\cos 32}$

$t = 3.71$

$y = -16t^2 + 135t\sin 32° + 1.5$

$y = -16(3.71)^2 + 135(3.71)\sin 32 + 1.5$

$= 46.68 > 10$

Y_C H.R.!