$0! = 1$ $1! = 1$ $2! = 2$ $3! = 6$ $4! = 24$ $5! = 120$	4. Given the series $\sum_{n=0}^{\infty}(-1)^n \frac{x^{2n}}{(2n)!}$ answer the following questions. a. List the first 6 terms of the series and the general term $\sum_{n=0}^{\infty}(-1)^n \frac{x^{2n}}{(2n)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \frac{x^{10}}{10!}$ $n=0 \quad n=1 \quad n=2$ b. Use the Ratio Test to determine the Interval of Convergence of the series. $\lim_{n\to\infty}\left\lvert \frac{(-1)^{n+1}x^{2n+2}}{(2n+2)!}\cdot\frac{(2n)!}{(-1)^n x^{2n}}\right\rvert = \left\lvert \frac{x^2}{(2n+2)(2n+1)}\right\rvert = 0 < 1$ $-\infty < x < \infty$ c. Enter the first 6 terms into y_1 of your calculator. Use $X[-\pi,\pi]_1$ and $Y[-1,1]$ as your window. d. What function does it look like the series represents? That function is the sum of this series. e. What would happen to the graphs if the first 10 terms of the series are entered into y_1.
$\frac{4}{4!} = \frac{4}{4\cdot3\cdot2\cdot1}$	f. Take the derivative of $\sum_{n=0}^{\infty}(-1)^n \frac{x^{2n}}{(2n)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots + (-1)^n \frac{x^{2n}}{(2n)!} + \cdots = \cos x$ $\frac{d}{dx}\left(\sum \frac{(-1)^n x^{2n}}{(2n)!}\right) = \sum_{n=0}^{\infty} \frac{(-1)^n (2n) x^{2n-1}}{(2n)!} = 0 - \frac{2x}{2!} + \frac{4x^3}{4!} - \cdots + (-1)^n \frac{(2n)x^{2n-1}}{(2n)!}$ power series $= \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n-1}}{(2n-1)!} = x + \frac{x^3}{6} - \cdots + \frac{(-1)^n x^{2n-1}}{(2n-1)!}$
$\frac{2n}{2n!} = \frac{\cancel{2n}}{\cancel{2n}\cdot(2n-1)(2n-2)\cdots}$	i. Take the anti-derivative of $\sum_{n=0}^{\infty}(-1)^n \frac{x^{2n}}{(2n)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots + (-1)^n \frac{x^{2n}}{(2n)!} + \cdots = \cos x$ $\int \sum \frac{(-1)^n x^{2n}}{(2n)!} = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)(2n)!} = x - \frac{1}{3}\cdot\frac{x^3}{2!} + \frac{1}{5}\cdot\frac{x^5}{4!} \cdots \frac{(-1)^n x^{2n+1}}{(2n+1)(2n)!}$ $= \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots + \frac{(-1)^n x^{2n+1}}{(2n+1)!}$

5. Given the series $\sum_{n=0}^{\infty}(-1)^n \frac{x^{2n+1}}{(2n+1)!}$ answer the following questions.

a. List the first 6 terms of the series and the general term

$$\sum_{n=0}^{\infty}(-1)^n \frac{x^{2n+1}}{(2n+1)!} =$$

b. Use the Ratio Test to determine the Interval of Convergence of the series.

c. Enter the first 6 terms into y_1 of your calculator. Use $X[-\pi,\pi]_1$ and $Y[-1,1]$ as your window.

d. What function does it look like the series represents? That function is the sum of this series.

e. Take the derivative of

$$\sum_{n=0}^{\infty}(-1)^n \frac{x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots - (-1)^n \frac{x^{2n+1}}{(2n+1)!} + \cdots = \underline{\sin x}$$

$$\frac{d}{dx} = \sum \frac{(-1)^n (2n+1) x^{2n}}{(2n+1)!} = 1 - \frac{3x^2}{3!} + \frac{5x^4}{5!} - \cdots + \frac{(-1)^n (2n+1) x^{2n}}{(2n+1)!}$$

$$= \sum \frac{(-1)^n x^{2n}}{(2n)!} = 1 - \frac{x^2}{2} + \frac{x^4}{4} - \cdots + \frac{(-1)^n x^{2n}}{(2n)!} = \cos x$$

f. Compare the derivative of the series in part f to the series you found in problem 4a.

g. Take the anti-derivative of

$$\sum_{n=0}^{\infty}(-1)^n \frac{x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots - (-1)^n \frac{x^{2n+1}}{(2n+1)!} + \cdots = \sin x$$

$$\sum_{n=0} \frac{(-1)^n x^{2n+2}}{(2n+2)!} = \frac{1}{2}x^2 - \frac{1}{4}\cdot\frac{x^4}{3!} + \frac{1}{6}\frac{x^6}{5!} = -\cos x + C$$

$$\frac{x^2}{2} - \frac{x^4}{4!} + \frac{x^6}{6!} + \cdots$$

1. Find the first 4 terms of the series
2. Write the rule for the series
3. Find the interval of convergence
4. Take the derivative of the function and series
5. Take the anti-derivative of the function and the series

3) $f(x)=\sin\left(x^2\right)$

$$f(x) = \sin x = \sum \frac{(-1)^n x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots + \frac{(-1)^n x^{2n+1}}{(2n+1)!}$$

$$f(x) = \sin(x^2) = \sum \frac{(-1) x^{2(2n+1)}}{(2n+1)!} = x^2 - \frac{(x^2)^3}{3!} + \frac{(x^2)^5}{5!} - \cdots +$$

$$f(x) = \sin(x^2) = \sum \frac{(-1)^n x^{4n+2}}{(2n+1)!} = x^2 - \frac{x^6}{3!} + \frac{x^{10}}{5!} - \cdots + \frac{(-1)^n x^{4n+2}}{(2n+1)!}$$

4) $f(x)=\underline{\underline{x^2}}\cos\left(x^3\right)$

$$f(x) = \cos x = \sum \frac{(-1)^n x^{2n}}{(2n)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots + \frac{(-1)^n x^{2n}}{(2n)!}$$

$$f(x) = \cos(x^3) = \sum \frac{(-1)^n (x^3)^{2n}}{(2n)!} = 1 - \frac{(x^3)^2}{2!} + \frac{(x^3)^4}{4!} - \cdots + \frac{(-1)^n (x^3)^{2n}}{(2n)!}$$

$$= \sum \frac{(-1)^n x^{6n}}{(2n)!} = 1 - \frac{x^6}{2!} + \frac{x^{12}}{4!} - \cdots \frac{(-1)^n x^{6n}}{(2n)!}$$

$$f(x) = x^2\cos(x^3) = \sum \frac{(-1)^n x^2 \cdot x^{6n}}{(2n)!} = x^2 - \frac{x^2 \cdot x^6}{2!} + \frac{x^2 \cdot x^{12}}{4!} - \cdots \frac{(-1)^n x^2 \cdot x^{6n}}{(2n)!}$$

$$= \sum \frac{(-1)^n x^{6n+2}}{(2n)!} = x^2 - \frac{x^8}{2!} + \frac{x^{14}}{4!} - \cdots \frac{(-1)^n x^{6n+2}}{(2n)!}$$