

Determine if the series converges or diverges using either the nth term test or recognizing if the series is geometric.

A) $\sum_{n=1}^{\infty} \frac{5n+2}{3n-1}$ Diverges

$$\lim_{n \rightarrow \infty} \frac{5n+2}{3n-1} = \frac{5}{3} \neq 0$$

B) $\sum_{n=1}^{\infty} \left(\frac{2}{5^n}\right) = \sum_{n=1}^{\infty} 2 \left(\frac{1}{5}\right)^n$ converges
 $|r| < 1$

$$\lim_{n \rightarrow \infty} \frac{2}{5^n} = 0$$

$$r = \frac{1}{5}$$

C) $\sum_{n=1}^{\infty} \left(\frac{4^n}{50}\right)$ Geometric OR $r = 4$ $|r| > 1$ diverges

$$\lim_{n \rightarrow \infty} \frac{4^n}{50} = \infty \neq 0 \text{ Diverges}$$

$$7! = 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$$

D) $\sum_{n=1}^{\infty} \left(\frac{2n!}{5n!+3}\right)$ Diverge

$$n! = n(n-1)(n-2)(n-3)\dots$$

$$\lim_{n \rightarrow \infty} \frac{2n!}{5n!+3} = \frac{2}{5} \neq 0$$

What you'll Learn About
 The Integral Test/P-Series Test/Comparison Test

$$\lim_{n \rightarrow \infty} \frac{1}{n^2} = 0$$

$$\lim_{n \rightarrow \infty} \frac{1}{n} = 0$$

Integral Test \rightarrow Converges b/c $\int_1^{\infty} \frac{1}{x^2}$ converges

A) $\sum_{n=1}^{\infty} \frac{1}{n^2}$

$$\lim_{b \rightarrow \infty} \int_1^b \frac{1}{x^2} = \int_1^b x^{-2} dx = -x^{-1} \Big|_1^b = -\frac{1}{x} \Big|_1^b$$

$$\lim_{b \rightarrow \infty} \left[-\frac{1}{b} - (-1) \right] = 1$$

Area Converges to 1

B) $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges b/c $\int_1^{\infty} \frac{1}{x}$ diverges

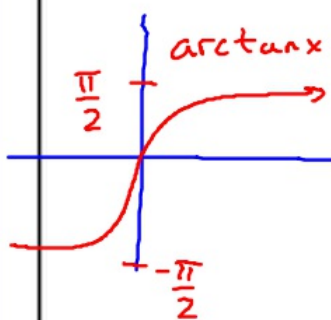
$$\lim_{b \rightarrow \infty} \int_1^b \frac{1}{x} dx = \ln x \Big|_1^b = \ln b - \ln 1 = \infty$$

Area Diverges

C) $\sum_{n=1}^{\infty} \frac{1}{n^{1/3}}$ diverges because $\int_1^{\infty} x^{-1/3}$ diverges

$$\lim_{b \rightarrow \infty} \int_1^b x^{-1/3} dx = \frac{3}{2} x^{2/3} \Big|_1^b = \frac{3}{2} b^{2/3} - \frac{3}{2} = \infty$$

diverges



$$D) \sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$$

$p = \frac{3}{2} > 1$ converges

$$E) \sum_{n=1}^{\infty} \frac{1}{n^2+9} \rightarrow \text{Converges b/c } \int_1^{\infty} \frac{1}{x^2+9} \text{ converges}$$

$$\lim_{b \rightarrow \infty} \int_1^b \frac{1}{x^2+9} dx = \left. \frac{1}{3} \arctan \frac{x}{3} \right|_1^b = \frac{1}{3} \arctan \frac{b}{3} - \frac{1}{3} \arctan \frac{1}{3}$$

$$= \frac{1}{3} \left(\frac{\pi}{2} \right) - \frac{1}{3} \arctan \frac{1}{3}$$

converges

P-Series Test

$$A) \sum_{n=1}^{\infty} \frac{1}{n^2}$$

$p = 2 > 1$ converges

$$B) \sum_{n=1}^{\infty} \frac{1}{n}$$

$p = 1 \leq 1$ diverges (harmonic)

$$C) \sum_{n=1}^{\infty} \frac{1}{n^{1/3}}$$

$p = \frac{1}{3} \leq 1$ diverges

$$D) \sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$$

$p = \frac{3}{2} > 1$ converges

$$E) \sum_{n=1}^{\infty} \frac{1}{n^{5/2}}$$

$p = \frac{5}{2} > 1$ converges

$$F) \sum_{n=1}^{\infty} \frac{1}{n^{1/2}}$$

$p = \frac{1}{2} \leq 1$ diverges