Page 17

Create the 3rd order Maclaurin Series for f(x) = arctan(x) by using the Taylor Polynomial process

Create the Maclaurin Series for f(x) =sinx by using the Taylor Polynomial process (5th order)

$f(x) = \sin x$ $\qquad f(0) = 0$

$f'(x) = \cos x$ $\qquad f'(0) = 1$

$f''(x) = -\sin x$ $\qquad f''(0) = 0$

$f^{3}(x) = -\cos x$ $\qquad f'''(0) = -1$

$f^{4}(x) = \sin x$ $\qquad f''''(0) = 0$

$f^{5}(x) = \cos x$ $\qquad f^{5}(0) = 1$

$$P_5(x) = x - \frac{x^3}{3} + \frac{x^5}{5!}$$

$f^5(x) = 1$ $\quad$ $f^{13}(x) = 1$

Write the first four terms for f(x) = sin(x)

$$x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - \frac{x^{11}}{11!} + \frac{x^{13}}{13!}$$

Find each of the following

1. $f^{(5)}(0) =$ 1 $\quad$ 2. $f^{(13)}(0) =$ 1 $\quad$ 3. $f^{(23)}(0) =$ −1

$4n+1$ positive derivatives

$4n+3$ neg derivatives

$f^{99}(0) = -1$

$$4n+3 = 99$$
$$-3 \quad -3$$
$$4n = 96$$
$$n = 24$$

$f^4(x)$

Write the first four terms for f(x) = sin(x^4)

$$x^4 - \frac{x^{12}}{3!} + \frac{x^{20}}{5!} - \frac{x^{28}}{7!}$$

Find each of the following

$f(x) = \sin(x^4)$

$f'(x) = \cos(x^4)\cdot 4x^3$

$f''(x) =$

1. $f^{(4)}(0) = 4!$ $\quad$ 2. $f^{(12)}(0) =$ $\quad$ 3. $f^{(20)}(0) = \boxed{\frac{20!}{5!} = \frac{1}{5!}\cdot 20!}$

4th derivative in front of 4th power

$$\frac{f^4(x)\,x^4}{4!} = \frac{(4!)\,x^4}{4!}$$

simplified

$$\frac{-x^{12}}{3!} = \frac{f^{12}(0)\,x^{12}}{12!}$$

$$\frac{-1}{3!} = \frac{f^{12}(0)}{12!}$$

$$\boxed{\frac{-(12!)}{3!} = f^{12}(0)}$$

$$\frac{x^{20}}{5!} = \frac{f^{20}(0)\,x^{20}}{20!}$$

$$\frac{1}{5!} = \frac{f^{20}(0)}{20!}$$

$$\boxed{\frac{20!}{5!} = f^{20}(0)}$$

	$\text{If } g(x) = \cdots \frac{1x^{12}}{12} \cdots \text{ find } f^{12}(0) = \frac{1}{12} \cdot 12! = 11!$ $\text{If } g(x) = \cdots \frac{1(x-3)^{20}}{10} \cdots \text{ find } f^{20}(3) = \frac{1}{10} \cdot 20! = \frac{20!}{10}$

① Take derivatives
② Plug in center
③ Build polynomial

Find the 3rd order Taylor Polynomial centered at x = 2

18) $f(x)=\frac{1}{x} = x^{-1}$ $\quad f(2)=\frac{1}{2}$

$f'(x) = -x^{-2} = -\frac{1}{x^2}$ $\quad f'(2) = \frac{-1}{4}$

$f''(x) = 2x^{-3} = \frac{2}{x^3}$ $\quad f''(2) = \frac{2}{8} = \frac{1}{4}$

$f'''(x) = -6x^{-4} = \frac{-6}{x^4}$ $\quad f'''(2) = \frac{-6}{16} = \frac{-3}{8}$

$$P_3(x-2) = \frac{\frac{1}{2}(x-2)^0}{0!} - \frac{\frac{1}{4}(x-2)^1}{1!} + \frac{\frac{1}{4}(x-2)^2}{2!} - \frac{\frac{3}{8}(x-2)^3}{3!}$$

$$\boxed{f(x) = \frac{1}{x}} \simeq P_3(x-2) = \boxed{\frac{1}{2} - \frac{1}{4}(x-2) + \frac{1}{8}(x-2)^2 - \frac{1}{16}(x-2)^3}$$

19) $f(x)=\sin x$ at $x=\frac{\pi}{4}$ center

$f(x) = \sin x$ $\quad f\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$

$f'(x) = \cos x$ $\quad f'\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$

$f''(x) = -\sin x$ $\quad f''\left(\frac{\pi}{4}\right) = -\sqrt{2}/2$

$f^3(x) = -\cos x$ $\quad f^3\left(\frac{\pi}{4}\right) = -\sqrt{2}/2$

$$P_3\left(x-\frac{\pi}{4}\right) = \frac{\frac{\sqrt{2}}{2}\left(x-\frac{\pi}{4}\right)^0}{0!} + \frac{\frac{\sqrt{2}}{2}\left(x-\frac{\pi}{4}\right)^1}{1!} - \frac{\frac{\sqrt{2}}{2}\left(x-\frac{\pi}{4}\right)^2}{2!} - \frac{\frac{\sqrt{2}}{2}\left(x-\frac{\pi}{4}\right)^3}{3!}$$

$$= \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\left(x-\frac{\pi}{4}\right) - \frac{\sqrt{2}}{4}\left(x-\frac{\pi}{4}\right)^2 - \frac{\sqrt{2}}{12}\left(x-\frac{\pi}{4}\right)^3$$

	Sometimes if your center is not at zero, you do not need to build the polynomial. You can use the MacLaurin series and Substitution. Construct the polynomial for $f(x) = e^{x-1}$ centered at x =1 using derivatives. Construct the polynomial for $f(x) = e^{x-1}$ centered at x = 1 using a MacLaurin series.