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What you'll Learn About
How to build a polynomial using derivatives

Given the values of the following, construct the 6[th] degree Taylor Polynomial centered at x = 0

$P(0) = 7$
$P'(0) = 3$
$P''(0) = 9$
$P'''(0) = 15$
$P^4(0) = 6$
$P^5(0) = 4$
$P^6(0) = 12$

$P(x) = a + bx + cx^2 + dx^3 + ex^4 + fx^5 + gx^6$

$P(0) = a$ $a = 7$

$P'(x) = b + 2cx + 3dx^2 + 4ex^3 + 5fx^4 + 6gx^5$

$3 = b$

$P''(x) = 2c + 6dx + 12ex^2 + 20fx^3 + 30gx^4$

$P''(0) = 2c$ $9 = 2c$ $c = \frac{9}{2}$

$P'''(x) = 6d + 24ex + 60fx^2 + 120gx^3$

$15 = 6d$ $d = \frac{15}{6}$

$P^4(x) = 24e + 120fx + 360gx^2$

$6 = 24e$ $e = \frac{6}{24}$

$P^5(x) = 120f + 720gx$ $4 = 120f$ $f = \frac{4}{120}$

$P^6(x) = 720g$ $12 = 720g$ $g = \frac{12}{720}$

$$\frac{(\text{value of derivative})\,x^n}{n!}$$

$$P_6(x) = 7 + 3x + \frac{9}{2}x^2 + \frac{15}{3!}x^3 + \frac{6}{4!}x^4 + \frac{4}{5!}x^5 + \frac{12}{6!}x^6$$

What would be the next 2 terms if $P^7(0) = 22$ and $P^8(0) = 50$?

7th derivative

$$\frac{22x^7}{7!} + \frac{50x^8}{8!}$$

$5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

Given the values of the following, construct the 4th degree Taylor Polynomial centered at x = 0

1. $P(0)=2$ $P'(0)=5$ $P''(0)=8$ $P'''(0)=11$ $P^4(0)=14$

$$P_4(x) = \frac{2x^0}{0!} + \frac{5x^1}{1!} + \frac{8x^2}{2!} + \frac{11x^3}{3!} + \frac{14x^4}{4!}$$

$$P_4(.2) = 2 + 5(.2) + \frac{8(.2)^2}{2!}$$

2. $P(0)=5$ $P'(0)=-2$ $P''(0)=7$ $P'''(0)=-4$ $P^4(0)=10$

$$P_5(x) = \frac{5x^0}{0!} - \frac{2x^1}{1!} + \frac{7x^2}{2!} - \frac{4x^3}{3!} + \frac{10x^4}{4!}$$

Given the values of the following, construct the 4th degree Taylor Polynomial centered at x = 2

3. $P(2)=2$ $P'(2)=5$ $P''(2)=8$ $P'''(2)=11$ $P^4(2)=14$

$$P_4(x-2) = \frac{2(x-2)^0}{0!} + \frac{5(x-2)^1}{1!} + \frac{8(x-2)^2}{2!} + \frac{3(x-2)^3}{3!} + \frac{14(x-2)^4}{4!}$$

$$P_4(x-2) = 2 + 5(x-2) + 4(x-2)^2 + \frac{1}{2}(x-2)^3 + \frac{7}{12}(x-2)^4$$

Given the values of the following, construct the 4th degree Taylor Polynomial centered at x = -2

4. $P(-2)=5$ $P'(-2)=-2$ $P''(-2)=7$ $P'''(-2)=-4$ $P^4(-2)=10$

$$P_4(x+2) = 5 - 2(x+2) + \frac{7(x+2)^2}{2} - 4\frac{(x+2)^3}{3!} + \frac{10(x+2)^4}{4!}$$

3^{rd} order = 3^{rd} degree (Take 3 derivatives)

centered at $x=0$

	Create the Maclaurin Series for f(x) = e^x by using the Taylor Polynomial process $f(x)=e^x \quad f(0)=e^0=1$ $f'(x)=e^x \quad f'(0)=1$ $f''(x)=e^x \quad f''(0)=1$ $f'''(x)=e^x \quad f'''(0)=1$
$e^x \approx$	$P_3(x)=\frac{1x^0}{0!}+\frac{1x^1}{1!}+\frac{1x^2}{2!}+\frac{1x^3}{3!}$ $\boxed{P_3(x)=1+x+\frac{x^2}{2!}+\frac{x^3}{3!}}$ Create the Maclaurin Series for f(x) =sinx by using the Taylor Polynomial process