CALCULUS: Graphical, Numerical, Algebraic by Finney, Demana, Watts and Kennedy
Chapter 9: Convergence of a Series

What you'll Learn About
What a geometric series is and whether or not the series converges or diverges
The nth term test for divergence

Given the first 4 terms of the Geometric Series: a) Write the general term of the series, b) Write the power series and c) Find the sum of the series, if possible

A) $4 + 8 + 16 + 32 + \ldots + (4)(2)^n + \ldots = \sum_{n=0}^{\infty} (4)(2)^n$

$S_1 = 4$  $S_3 = 28$
$S_2 = 12$  $S_4 = 60$

General Term $a_n = (a_1)(r)^n$

Power Series

$B) \frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \ldots + \left(\frac{1}{4}\right)^n + \ldots = \sum_{n=0}^{\infty} \left(\frac{1}{4}\right)^n$

$S_1 = 1$  $S_4 = 0.196$
$S_2 = 0.75$  $S_5 = 0.800$
$S_3 = 0.8125$  $S_6 = 0.799$

$S = \frac{1}{1-\left(\frac{1}{4}\right)} = \frac{4}{3}$

Sum Converges to $\frac{4}{3}$

C) $5 + 15 + 45 + 135 + \ldots$

$S_1 = 5$  $S_4 = 500$
$S_2 = 20$  $S_5 = 1300$
$S_3 = 60$  $S_6 = 5000$

$S = \frac{1}{1 - \left(\frac{3}{5}\right)} = \frac{5}{2}$

Sum Converges because $r = \frac{3}{5}$ is between $-1$ and $1$

D) $1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \ldots$

$r = -\frac{1}{2}$

$S = \frac{1}{1 - \left(-\frac{1}{2}\right)} = \frac{2}{3}$

Converges by $|r| < 1$

Sum Diverges because $r = \frac{3}{5}$ is not between $-1$ and $1$
Given the first 4 terms of the Geometric Series; a) Write the general
term of the series, b) Write the power series, c) Find the equation for
the sum of the series, and d) the Interval and Radius of Convergence of
the series.

\[
A) \quad 1 + 5x + 25x^2 + 125x^3 + \cdots + (1) \left(5x\right)^n = \sum_{n=0}^{\infty} (5x)^n
\]

\[
x=1 \quad 1 + 5 + 25 + 125 \quad \cdots
\]

\[
x=\frac{1}{10} \quad 1 + \frac{5}{10} + \frac{25}{100} + \frac{125}{1000}
\]

**Interval of Convergence**

\[-1 \leq r \leq 1\]

\[-\frac{1}{5} \leq 5x \leq \frac{1}{5}\]

**Radius of Convergence**

\[-\frac{1}{5} \leq x \leq \frac{1}{5}\]

**Centered at** \(x=0\)

**B) 1-(x-1)+(x-1)^2-(x-1)^3+\cdots+(-1)^n(x-1)^n = \sum_{n=0}^{\infty} (-1)^n(x-1)^n**

\[r = -\left(x-1\right)\]

**I.O.C.**

\[-1 \leq r \leq 1\]

\[-\frac{1}{1} \leq -\left(x-1\right) \leq \frac{1}{1}\]

\[-1 \leq x-1 \leq 1\]

\[1 > x-1 > -1\]

\[+1 \quad +1\]

\[2 > x > 0\]

**Centered at** \(x=1\)

**R.O.C.**

\[= 1\]
Determine if the geometric series converges or diverges. If the series converges, find the value of the series.

1) \( \sum_{n=0}^{\infty} 3 \left( \frac{3}{2} \right)^n \)

Diverges \( |r| > 1 \quad r = \frac{3}{2} \)

2) \( \sum_{n=1}^{\infty} \frac{9}{4} \left( \frac{1}{4} \right)^n \)

Converge \( r = \frac{1}{4} \quad |r| < 1 \)

\[
S = \frac{a_1}{1-r} = \frac{\frac{9}{16}}{1 - \frac{1}{4}} = \frac{\frac{9}{16}}{\frac{3}{4}} = \frac{3}{4}
\]

12) \( \sum_{n=1}^{\infty} \left( \frac{x^2}{x^2 + 4} \right)^n \)