What you'll Learn About
What a geometric series is and whether or not the series converges or diverges
The nth term test for divergence

Given the first 4 terms of the Geometric Series; a) Write the general term of the series, b) Write the power series and c) Find the sum of the series, if possible

Geometric
common ratio
$r = 2$
General Term
$a_n = (a_1)(r)^n$
1st term, common ratio

$-1 < r < 1$ → $|r| < 1$

$S = \frac{a_1}{1-r}$

$|r| > 1$ ← Sum Diverges because r is not between -1 and 1

A) $4+8+16+32+\cdots+(4)(2)^n+\cdots = \sum_{n=0}^{\infty}(4)(2)^n$

$n=0$ ↑ general term

$S_1 = 4$ $S_3 = 28$
$S_2 = 12$ $S_4 = 60$

Power Series

Sum Diverges

B) $1-\frac{1}{4}+\frac{1}{16}-\frac{1}{64}+\cdots+(1)\left(\frac{-1}{4}\right)^n+\cdots = \sum_{n=0}^{\infty}(-1)^n\left(\frac{1}{4}\right)^n$

$(-1)^n(1)\left(\frac{1}{4}\right)^n$

$S_1 = 1$ $S_4 = .796$ $S_7 = .80004$
$S_2 = .75$ $S_5 = .800$
$S_3 = .8125$ $S_6 \doteq .799$

Sum Converges to .8

C) $5+15+45+135+\cdots$

$5+15+45+135+\cdots+(5)(3)^n+\cdots = \sum_{n=0}^{\infty}5(3)^n$

$S = \frac{1}{1-\left(\frac{-1}{4}\right)} = \frac{1}{\left(\frac{5}{4}\right)} = \frac{4}{5}$

D) $1-\frac{1}{2}+\frac{1}{4}-\frac{1}{8}+\cdots$

$r = -\frac{1}{2}$ ← $1-\frac{1}{2}+\frac{1}{4}-\frac{1}{8}+\cdots+(-1)^n\left(\frac{1}{2}\right)^n+\cdots = \sum_{n=0}^{\infty}(-1)^n\left(\frac{1}{2}\right)^n$

Converges b/c $|r| < 1$ $S = \frac{1}{1-\left(\frac{-1}{2}\right)} = \frac{1}{\frac{3}{2}} = \frac{2}{3}$

Given the first 4 terms of the Geometric Series; a) Write the general term of the series, b) Write the power series, c) Find the equation for the sum of the series, and d) the Interval and Radius of Convergence of the series.

$r = 5x$

A) $1+5x+25x^2+125x^3+\cdots+(1)(5x)^n+\cdots=\sum_{n=0}^{\infty}(5x)^n$

Diverges $x=1$ $1+5+25+125\cdots$

Converges $x=\frac{1}{10}$ $1+\frac{5}{10}+\frac{25}{100}+\frac{125}{1000}$

$$S=\frac{1}{1-5x}$$

Radius of Convergence $=\frac{1}{5}$

Interval of Convergence

$-1<r<1$

$-\frac{1}{5}<\frac{5}{5}x<\frac{1}{5}$

$-\frac{1}{5}<x<\frac{1}{5}$

centered at $x=0$

B) $1-(x-1)+(x-1)^2-(x-1)^3+\cdots+(-1)^n(x-1)^n+\cdots=\sum_{n=0}^{\infty}(-1)^n(x-1)^n$

$r=-(x-1)$

$$S=\frac{1}{1-(-(x-1))}$$

I.O.C $-1<r<1$

$\frac{-1}{-1}<\frac{-(x-1)}{-1}<\frac{1}{-1}$

$1>x-1>-1$

$+1 \quad +1 \quad +1$

$2>x>0$

Centered at $x=1$

R.O.C $=1$

Find the Sum

Determine if the geometric series converges or diverges. If the series converges find the value of the series.

1) $\sum_{n=0}^{\infty} 3\left(\frac{3}{2}\right)^n$ diverges $|r| > 1$ $r = \frac{3}{2}$

2) $\sum_{n=1}^{\infty} \frac{9}{4}\left(\frac{1}{4}\right)^n$ converge $r = \frac{1}{4}$ $|r| < 1$

$\frac{9}{4} \cdot \frac{1}{4} + \frac{9}{4} \cdot \frac{1}{16}$

$\frac{9}{16} + \frac{9}{64}$

$S = \frac{a_1}{1-r} = \frac{\frac{9}{16}}{1-\frac{1}{4}} = \frac{\left(\frac{9}{16}\right)}{\left(\frac{3}{4}\right)}$

$\frac{9}{16} \cdot \frac{4}{3} = \frac{3}{4}$

12) $\sum_{n=1}^{\infty} \left(\frac{x^2}{x^2+4}\right)^n$