p. 527 #60
Let \( f(x) = \frac{1}{x-2} \) at \( x = 3 \).

a. Write the first 4 terms and the general term of the Taylor Series generated by \( f(x) \) at \( x = 3 \).

\[
\begin{align*}
  f(x) &= (x-2)^{-1} \\
  f'(x) &= -(x-2)^{-2} \\
  f''(x) &= 2(x-2)^{-3} \\
  f'''(x) &= -6(x-2)^{-4}
\end{align*}
\]

\[ f(3) = 1 \quad \text{general term} \]
\[ f'(3) = -1 \] \[ f''(3) = 2 \] \[ f'''(3) = -6 \]

b. Use the result in part (a) to find the fourth order polynomial and the general term of the series generated by \( \ln|x-2| \) at \( x = 3 \).

\[
\int f(x) = (x-3) - \frac{1}{2} (x-3)^2 + \frac{1}{3} (x-3)^3 - \frac{1}{4} (x-3)^4
\]

\[ x = 3.5 \quad -2.5 - \frac{1}{2} (2.5)^2 + \frac{1}{3} (2.5)^3 - \frac{1}{4} (2.5)^4 \]

\[ \ln|3.5-2| \]

\[ n \geq 5 \]

\[ n = 5 \]

\[ a_5 = \frac{1}{3} (3.5-3)^5 \]

\[ \text{error bound} \leq \frac{1}{5} (x-3) \]

83. The Taylor Series for \( \ln x \), centered at \( x = 1 \), is \( \sum_{n=1}^\infty \frac{(-1)^{n+1}(x-1)^n}{n} \). Let \( f \) be the function given by the sum of the first three nonzero terms of this series. The maximum value of \( |\ln x - f(x)| \) for \( .3 \leq x \leq 1.7 \) is

(A) .030 \quad (B) .039 \quad (C) .145 \quad (D) .153 \quad (E) .529
2011 BC6

Let $f(x) = \sin(x^2) + \cos x$.

a. Write the first four nonzero terms of the Taylor series for $\sin x$ about $x = 0$, and write the first four nonzero terms of the Taylor series for $\sin(x^2)$ about $x = 0$.

b. Write the first four nonzero terms of the Taylor series for $\cos x$ about $x = 0$. Use this series and the series for $\sin(x^2)$, found in part a, to write the first four nonzero terms of the Taylor series for $f(x)$ about $x = 0$.

$$f(x) = \sin(x^2) + \cos x$$

$$f(x) = x^2 - \frac{x^6}{3!} + \frac{x^{10}}{5!} - \frac{x^{14}}{7!} + \left(1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!}\right)$$

$$= 1 + \left(x^2 - \frac{x^2}{2!}\right) + \frac{x^4}{4!} + \left(-\frac{x^6}{3!} - \frac{x^6}{6!}\right)$$

c. Find the value of $f^{(6)}(0)$.

$$\frac{f^{(6)}(0)}{6!} = \frac{-121}{6!}$$

$$f^{(6)}(0) = -121$$

d. Let $f(x) = \sin(x^2) + \cos x$. The graph of $y = |f^{(5)}(x)|$ is shown.

Let $p_4(x)$ be the fourth degree Taylor polynomial for $f$ about $x = 0$. Using information from the graph of $y = |f^{(5)}(x)|$, shown above, show that

$$\left| p_4\left(\frac{1}{4}\right) - f\left(\frac{1}{4}\right) \right| < \frac{1}{3000}. $$
2004 BC6

Let \( f \) be the function given by \( f(x) = \sin\left(5x + \frac{\pi}{4}\right) \), and let \( P(x) \) be the third-degree Taylor polynomial for \( f \) about \( x = 0 \).

a) Find \( P(x) \).

b) Find the coefficient of \( x^2 \) in the Taylor series about \( x = 0 \).

c) Use the Lagrange error bound to show that \( \left| f\left(\frac{1}{10}\right) - P\left(\frac{1}{10}\right) \right| < \frac{1}{100} \).

d) Let \( G \) be the function given \( G(x) = \int_0^x f(t) \, dt \). Write the third-degree Taylor polynomial for \( G \) about \( x = 0 \).
\[ f(x) = \sum_{n=0}^{\infty} c_n x^n \]

**Interval of Convergence** | **Radius of Convergence**
--- | ---
\[ \frac{1}{1-x} = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + \ldots \] | (−1, 1) | 1
\[ e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots \] | (−∞, ∞) | ∞
\[ \sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} \] | (−∞, ∞) | ∞
\[ = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots \]
\[ \cos x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} \] | (−∞, ∞) | ∞
\[ = 1 + \frac{x^2}{2!} - \frac{x^4}{4!} + \frac{x^6}{6!} + \ldots \]
\[ \tan^{-1} x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1} \] | (−1, 1] | 1
\[ = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \ldots \]
\[ \ln(1+x) = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n} \] | (−1, 1] | 1
\[ = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots \]