What you'll Learn About
How to write terms given a power series
How to take the derivative and anti-derivative of a power series
Identifying important types of power series

1. Given the series $\sum_{n=0}^{\infty} x^n$ answer the following questions.

a. List the first 6 terms of the series and the general term

$$\sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + x^4 + x^5$$

$r = x$

b. Determine the Interval of Convergence of the series.

$-1 < x < 1$

c. Determine the function (sum) of the series (f(x) =)

$f(x) = \frac{1}{1-x}$

d. Enter the first 6 terms into y_1 and the function (sum) into y_2 of your calculator.

e. Set the x-values of your window to match your interval of convergence and the y-values from [0,10]. What do you notice about the 2 graphs?

They get closer together on the interval of convergence

f. What would happen to the graphs if the first 10 terms of the series are entered into y_1.

Sum

i. Take the derivative of $\boxed{\sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + \cdots x^n = \left(\frac{1}{1-x}\right) = (1-x)^{-1}}$

$$\sum_{n=0}^{\infty} n x^{n-1} = 0 + 1 + 2x + 3x^2 + \cdots n x^{n-1} = +(1-x)^{-2} = \frac{1}{(1-x)^2}$$

$x = 0$
sum = 1

j. Take the anti- derivative of $\sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + \cdots x^n = \frac{1}{1-x}$

$$\sum_{n=0}^{\infty} \frac{x^{n+1}}{n+1} = x + \frac{1}{2}x^2 + \frac{1}{3}x^3 + \frac{1}{4}x^4 + \cdots \frac{1x^{n+1}}{n+1} = -\ln|1-x|$$

What you'll Learn About
Taking derivatives and anti-derivatives of a power series

multiply by $-x^3$ each time

1. Find the first 4 terms of the series
2. Write the rule for the series
3. Find the interval of convergence
4. Take the derivative of the function and series
5. Take the anti-derivative of the function and the series

1st term = 1 $\quad r = -x^3$

Geometric Series

1) $f(x)=\dfrac{1}{1+x^3} = 1 - x^3 + x^6 - x^9 + \cdots (-1)^n x^{3n} + \cdots = \sum_{n=0}^{\infty} (-1)^n x^{3n}$

$\dfrac{-1}{-1} < \dfrac{-x^3}{-1} < \dfrac{1}{-1}$

$1 > x^3 > -1$

$\boxed{1 > x > -1}$

$\int \dfrac{1}{1+x^3} = x - \frac{1}{4}x^4 + \frac{1}{7}x^7 - \frac{1}{10}x^{10} + \cdots \dfrac{(-1)^n x^{3n+1}}{3n+1} + \cdots = \sum_{n=0}^{\infty} \dfrac{(-1)^n x^{3n+1}}{3n+1}$

$\dfrac{d}{dx}\left(\dfrac{1}{1+x^3}\right) = 0 - 3x^2 + 6x^5 - 9x^8 + \cdots (-1)^n (3n) x^{3n-1} = \sum (-1)^n (3n) x^{3n-1}$

1st term = x
$r = x^2$

I.O.C
$-1 < x^2 < 1$
$-1 < x < 1$

2) $f(x)=\dfrac{x}{1-x^2} = x^1 + x^3 + x^5 + x^7 + \cdots x^{2n+1} + \cdots \sum_{n=0}^{\infty} x^{2n+1}$

$\dfrac{d}{dx}\left(\dfrac{x}{1-x^2}\right) = 1 + 3x^2 + 5x^4 + 7x^6 + \cdots (2n+1) x^{2n} + \cdots = \sum_{n=0}^{\infty} (2n+1) x^{2n}$

$\int \dfrac{x}{1-x^2} = \frac{1}{2}x^2 + \frac{1}{4}x^4 + \frac{1}{6}x^6 + \frac{1}{8}x^8 + \cdots \dfrac{x^{2n+2}}{2n+2} + \cdots = \sum_{n=0}^{\infty} \dfrac{x^{2n+2}}{2n+2}$