

$$x = -1/3$$

$$\sum_{n=1}^{\infty} (-3)^{n-1} \left(\frac{1}{3}\right)^n$$

$$\sum_{n=1}^{\infty} \frac{(-3)^n \cdot (-3)^{-1} \left(\frac{1}{3}\right)^n}{n}$$

$\sum_{n=1}^{\infty} \frac{1}{3^n}$   
multiple harmonic  
diverges

$$x = +1/3$$

$$\sum_{n=1}^{\infty} (-1)^n \left(\frac{1}{3n}\right)$$

conditional convergence

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$n=1$   $n=2$   $n=3$   $n=4$

$$x - \frac{3}{2}x^2 + 3x^3 + \frac{(-3)^3 x^4}{4}$$

1. The Maclaurin series for a function  $f$  is given by

$$f(x) = \sum_{n=1}^{\infty} \frac{(-3)^{n-1}}{n} x^n = x - \frac{3}{2}x^2 + 3x^3 - \dots + \frac{(-3)^{n-1}}{n} x^n \dots \text{and converges to } f(x) \text{ for } |x| < R, \text{ where } R \text{ is the radius of convergence of the Maclaurin series.}$$

$$R = \frac{1}{3}$$

a) Use the Ratio Test to find  $R$

$$\lim_{n \rightarrow \infty} \left| \frac{(-3)^n x^{n+1}}{n+1} \cdot \frac{n}{(-3)^{n-1} x^n} \right| = \left| \frac{x}{(-3)^{-1}} \right| = |3x| < 1$$

$$-1 < 3x < 1$$

$$-\frac{1}{3} < x < \frac{1}{3}$$

b) Write the first four non-zero terms of the Maclaurin series for  $f'$ , the derivative of  $f$ . Express  $f'$  as a rational function for  $|x| < R$ .

$$f'(x) = 1 - 3x + 9x^2 - 27x^3$$

Sum of geo

$$f'(x) = \frac{1}{1 - (-3x)} = \frac{1}{1 + 3x}$$

c) Write the first four nonzero terms of the Maclaurin series for  $e^x$ . Use the Maclaurin series for  $e^x$  to write the third-degree polynomial for  $g(x) = e^x f(x)$  about  $x = 0$ .

$$f(x) = x - \frac{3}{2}x^2 + 3x^3 - \frac{27x^4}{4}$$

$$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{3!}$$

$$e^x f(x) = x - \frac{3}{2}x^2 + 3x^3 + \frac{1}{2}x^2 - \frac{3}{2}x^3 + \frac{1}{2}x^3$$

$$= x - \frac{1}{2}x^2 + 2x^3$$