What you'll Learn About
How to find the error for a series that alternates

$P(5) = 5$

$\arctan(5) = 1.37$

1. Give the first term of the series for f(x) = arctan(x) centered at x = 0

   $P(x) = x$

2. Find the approximation for P(.1) $= .1$

3. Find the f(.1) $= \arctan(.1) = .0996$ Actual

   Approximation

4. How accurate is the approximation. $|.0996 - .1| = .0003313475$

5. What is the value of the next term of the polynomial at x =.1

Next term $\frac{-x^3}{3}$ $\left|\frac{.1^3}{3}\right| = .000\overline{3}$

1. Give the first 2 terms of the series for f(x) = arctan(x) centered at x = 0

$P(x) = x - x^3/3$

2. Find the approximation for P(.1) $= .0996\overline{6}$

3. Find the f(.1) $= \arctan(.1) = .0996686525$

4. How accurate is the approximation. $|\arctan(.1) - P_2(.1)| = .000001985824$

5. What is the value of the next term of the polynomial at x =.1

Next Term $\frac{x^5}{5}$ $\left|\frac{.1^5}{5}\right| = .000002$

1. Give the first 3 terms of the series for f(x) = arctan(x) centered at x = 0

   $P(x) = x - x^3/3 + x^5/5$

2. Find the approximation for P(.1) $P(.1) = .0996686667$

3. Find the f(.1) $= .0996686525$

4. How accurate is the approximation. $|\arctan(.1) - P_3(.1)| = .00000001417505$

5. What is the value of the next term of the polynomial at x =.1

$\frac{-x^7}{7}$ $\left|\frac{.1^7}{7}\right| = .00000001428571429$

	1. Give the first 4 terms of the series for f(x) = arctan(x) centered at x = 0 $P_4(x) = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7}$ *2. Use the alternate estimation theorem to determine the error bound* $\lvert f(x) - P(x) \rvert \le R$ Actual Approximation Error Bound Next Term $= \frac{x^9}{9}$ $\quad \left\lvert \frac{.1^9}{9} \right\rvert = R$
$f(x) = \sin x$ $f'(x) = \cos x$ $f''(x) = -\sin x$ $f'''(x) = -\cos x$ $f^4(x) = \sin x$ $f^5(x) = \cos x$ $f^6(x) = -\sin x$	1. Give the first 4 terms of the series for f(x) = sin(x) centered at x = $\frac{\pi}{2}$ $x = \pi/2$ 1 0 -1 0 1 0 -1 $P(x) = 1 - \frac{(x-\frac{\pi}{2})^2}{2!} + \frac{(x-\frac{\pi}{2})^4}{4!} - \frac{(x-\frac{\pi}{2})^6}{6!}$ 2. Use the alternate estimation theorem to determine the error bound at x =1.6 $\lvert f(x) - P(x) \rvert \le R$ Next Term $= \frac{(x-\frac{\pi}{2})^8}{8!}$ $\quad R = \left\lvert \frac{(1.6 - \pi/2)^8}{8!} \right\rvert$

order = # of derivative = highest power

$x = 2$ $f(x) = x^{-1} \longrightarrow 1/2$ $f'(x) = -x^{-2} \rightarrow -1/4$ $f''(x) = 2x^{-3} \rightarrow 1/4$ $f'''(x) = -6x^{-4} \rightarrow -3/8$ $f^{4}(x) = 24x^{-5} \rightarrow 3/4$ $f^{5}(x) = -120x^{-6} = -\frac{120}{x^6}$	1. $f(x) = \frac{1}{x}$ centered at x = 2 a. Given the function, find the fourth order polynomial $P_4(x-2) = \frac{1}{2} - \frac{1}{4}(x-2) + \frac{\frac{1}{4}(x-2)^2}{2} - \frac{\frac{3}{8}(x-2)^3}{3!} + \frac{\frac{3}{4}(x-2)^4}{4!}$ b. Use the alternate estimation theorem to find a formula for the error bound $\lvert f(x) - P(x) \rvert \leq R$ ↑ Next Term $R = \left\lvert \frac{\frac{-120}{64}(x-2)^5}{5!} \right\rvert = \left\lvert \frac{f^{5}(2)(x-2)^5}{5!} \right\rvert$