	Determine which value the series converges to. (Determine the value of the series/Determine the sum of the series) $A)\ \sum_{n=0}^{\infty}\left(\frac{4}{5}\right)^n = 1 + \frac{4}{5} + \left(\frac{4}{5}\right)^2 + \cdots$ $f(x) = \frac{1}{1-\frac{4}{5}} = 5$ $B)\ \sum_{n=0}^{\infty}\frac{5^n}{n!} = 1 + \frac{5}{1} + \frac{5^2}{2!} + \frac{5^3}{3!} +$ $\sum_{n=0}^{\infty}\frac{5^n}{n!} = e^5$ $\sum_{n=0}^{\infty}\frac{x^n}{n!} = e^x$ $C)\ \sum_{n=0}^{\infty}\frac{(-1)^n\left(\frac{\pi}{4}\right)^{2n+1}}{(2n+1)!} = \sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$ $D)\ \sum_{n=0}^{\infty}\frac{(-1)^n\left(\frac{1}{2}\right)^{2n+1}}{2n+1} = \arctan\left(\frac{1}{2}\right)$

What you'll Learn About
Taking derivatives and anti-derivatives of a power series

1. Find the first 4 terms of the series
2. Write the rule for the series
3. Find the interval of convergence
4. Take the derivative of the series
5. Take the anti-derivative of the series

Geometric Series

1) $f(x)=\dfrac{1}{1+x^3}$

2) $f(x)=\dfrac{x}{1-x^2} = x + x^3 + x^5 + x^7 + \cdots + x^{2n+1} + \cdots = \sum_{n=0}^{\infty} x^{2n+1}$

1st term $= x$
$r = x^2$

I.O.C
$-1 < x^2 < 1$

$-1 < x < 1$

$\dfrac{d}{dx} \sum_{n=0}^{\infty} x^{2n+1} = \sum_{n=0}^{\infty} (2n+1)\, x^{2n}$

$\int \sum_{n=0}^{\infty} x^{2n+1} = \sum_{n=0}^{\infty} \dfrac{x^{2n+2}}{2n+2}$

1. Find the first 4 terms of the series
2. Write the rule for the series
3. Find the interval of convergence
4. Take the derivative of the series
5. Take the anti-derivative of the series

3) $f(x) = \sin(x^2)$

4) $f(x) = x^2 \cos(x^3)$

$$\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots - \frac{(-1)^n x^{2n}}{(2n)!} + \cdots = \sum \frac{(-1)^n x^{2n}}{(2n)!}$$

I.O.C

$-\infty < x < \infty$

$$\boxed{\cos(x^3) = 1 - \frac{(x^3)^2}{2!} + \frac{(x^3)^4}{4!} - \frac{(x^3)^6}{6!} + \cdots - \frac{(-1)^n (x^3)^{2n}}{(2n)!}}$$

$$\cos(x^3) = 1 - \frac{x^6}{2!} + \frac{x^{12}}{4!} - \frac{x^{18}}{6!} + \cdots \frac{(-1)^n x^{6n}}{(2n)!}$$

$$x^2 \cos(x^3) = x^2 - \frac{x^8}{2!} + \frac{x^{14}}{4!} - \frac{x^{20}}{6!} + \cdots - \frac{(-1)^n x^{6n+2}}{(2n)!} =$$

$$\boxed{\frac{d}{dx} \sum_{n=0}^{\infty} \frac{(-1)^n x^{6n+2}}{(2n)!} = \sum_{n=0}^{\infty} \frac{(-1)^n (6n+2) x^{6n+1}}{(2n)!}}$$

$$\int \sum_{n=0}^{\infty} \frac{(-1)^n x^{6n+2}}{(2n)!} = \sum_{n=0}^{\infty} \frac{(-1)^n x^{6n+3}}{(6n+3)(2n)!} + C$$