2a. Build the MaClaurin Series for $f(x)=\ln(1+x)$

$$P(x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} - \cdots + \frac{(-1)^{n+1}x^n}{n} = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}x^n}{n}$$

b. Determine the Interval of Convergence of the series.

$$\boxed{-1 < x \le 1}$$

$$\lim_{n\to\infty} \left| \frac{(-1)^{n+2}x^{n+1}}{n+1} \cdot \frac{n}{(-1)^{n+1}x^n} \right| = |x| < 1$$

$x=-1$

$$\sum \frac{(-1)^{n+1}(-1)^n}{n}$$

$$\sum (-1)^{2n+1}/n \quad \text{Diverges}$$

$x=1$

$$\sum \frac{(-1)^{n+1}}{n}$$

c. Take the derivative of the power series for $f(x)=\ln(1+x)$

$$\frac{d}{dx}\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x^n)}{n} = \sum \frac{(-1)^{n+1} n x^{n-1}}{n} = \sum_{n=1}^{\infty} (-1)^{n+1} x^{n-1}$$

d. Take the anti-derivative of the power series for $f(x)=\ln(1+x)$

$$\int \sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x^n)}{n} = \sum_{n=1}^{\infty} \frac{(-1)^{n+1} x^{n+1}}{n(n+1)} + C$$

3a. Build the MaClaurin Series for $\boxed{f(x) = \sin x}$ → 5th order

① Take Derivatives
② Plug in Center
③ Build Polynomial

$f(x) = \sin x \quad f(0) = 0$

$f'(x) = \cos x \quad f'(0) = 1$

$f''(x) = -\sin x \quad f''(0) = 0$

$f'''(x) = -\cos x \quad f'''(0) = -1$

$f^4(x) = \sin x \quad f^4(0) = 0$

$f^5(x) = \cos x \quad f^5(0) = 1$

$P(x) = \frac{0x^0}{0!} + \frac{1x^1}{1!} + \frac{0x^2}{2!} - \frac{1x^3}{3!} + \frac{0x^4}{4!} + \frac{1x^5}{5!}$

$P(x) = \frac{x^1}{1} - \frac{x^3}{3!} + \frac{x^5}{5!}$

$$\boxed{\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}}$$

b. Determine the Interval of Convergence of the series.

$$\lim_{n\to\infty} \left| \frac{(-1)^{n+1} x^{2n+3}}{(2n+3)!} \cdot \frac{(2n+1)!}{(-1)^n x^{2n+1}} \right| = 0 < 1$$

$-\infty < x < \infty$

c. Take the derivative of the power series for $f(x) = \sin x$

$$\sum_{n=0}^{\infty} \frac{(-1)^n (2n+1) x^{2n}}{(2n+1)!} = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$$

$$\frac{2n+1}{(2n+1)!} = \frac{\cancel{(2n+1)}}{\cancel{(2n+1)}(2n)(2n-1)}$$

d. Take the anti-derivative of the power series for $f(x) = \sin x$

	4a.	Build the MaClaurin Series for $f(x) = \cos x$
	b.	Determine the Interval of Convergence of the series.
	c.	Take the derivative of the power series for $f(x) = \cos x$
	d.	Take the anti-derivative of the power series for $f(x) = \cos x$

5a. Build the MaClaurin Series for $f(x) = e^x$

b. Determine the Interval of Convergence of the series.

c. Take the derivative of the power series for $f(x) = e^x$

d. Take the anti-derivative of the power series for $f(x) = e^x$

$P_3(x) = \frac{1}{1}x^1 - \frac{1}{3}x^3 + \frac{1}{5}x^5$

$f(0) = 0$ $f'(0) = 1$ $f''(0) = 0$ $f'''(0) = \frac{-2}{1} = -2$	6a. Build the MaClaurin Series for $f(x) = \arctan(x)$ $f(x) = \arctan x$ $f'(x) = \frac{1}{1+x^2} = (1+x^2)^{-1}$ $f''(x) = -(1+x^2)^{-2} \cdot 2x = \frac{-2x}{(1+x^2)^2}$ $f'''(x) = \frac{(1+x^2)^2(-2) - (-2x)[2(1+x^2)\cdot 2x]}{(1+x^2)^4}$ $P_3(x) = \frac{1x^1}{1!} - \frac{2x^3}{3!}$ $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1}$ b. Determine the Interval of Convergence of the series. $\lim_{n\to\infty} \left\| \frac{(-1)^{n+1} x^{2n+3}}{2n+3} \cdot \frac{(2n+1)}{(-1)^n x^{2n+1}} \right\| = \|x^2\| < 1$ $-1 < x^2 < \sqrt{1}$ $\boxed{-1 \leq x \leq 1}$ $\sum \frac{(-1)^n (-1)^{2n+1}}{2n+1} = \frac{(-1)^{3n+1}}{2n+1}$ c. Take the derivative of the power series for $f(x) = \arctan(x)$ d. Take the anti-derivative of the power series for $f(x) = \arctan(x)$

$f(x)=\sum_{n=0}^{\infty} c_n x^n$	INTERVAL OF CONVERGENCE	RADIUS OF CONVERGENCE
$\frac{1}{1-x}=\sum_{n=0}^{\infty} x^n = 1+x+x^2+x^3+\cdots$	$(-1,1)$	1
$e^x=\sum_{n=0}^{\infty}\frac{x^n}{n!}=1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\cdots$	$(-\infty,\infty)$	∞
$\sin x=\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{(2n+1)!} = x-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}+\cdots$	$(-\infty,\infty)$	∞
$\cos x=\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n}}{(2n)!} = 1-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}+\cdots$	$(-\infty,\infty)$	∞
$\tan^{-1} x=\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{2n+1} = x-\frac{x^3}{3}+\frac{x^5}{5}-\frac{x^7}{7}+\cdots$	$(-1,1]$	1
$\ln(1+x)=\sum_{n=1}^{\infty}(-1)^{n-1}\frac{x^n}{n} = x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\cdots$	$(-1,1]$	1