

$|r| > 1$  diverge

$|r| < 1$  converged

$$\text{Sum} = f(x) = \frac{a}{1-r}$$

$a$ : 1st term

Determine if the geometric series converges or diverges. If the series converges find the value of the series.

$$1) \sum_{n=0}^{\infty} 3\left(\frac{3}{2}\right)^n = 3 + \frac{9}{2} + \frac{27}{4} + \dots$$

common ratio:  $r = \frac{3}{2} > 1$  diverges  
 $|r| = \frac{3}{2} > 1$

$$2) \sum_{n=1}^{\infty} \frac{9}{4}\left(\frac{1}{4}\right)^n = \frac{9}{16} + \frac{9}{64} + \frac{9}{256} + \dots$$

$r = \frac{1}{4} < 1$  converges  
 $|r| = \frac{1}{4} < 1$

$$\text{Sum} = \frac{\frac{9}{16}}{1 - \frac{1}{4}} = \frac{\left(\frac{9}{16}\right)}{\left(\frac{3}{4}\right)} = \frac{36}{48} = \frac{3}{4}$$

$$12) \sum_{n=1}^{\infty} \left(\frac{x^2}{x^2+4}\right)^n = \frac{x^2}{x^2+4} + \left(\frac{x^2}{x^2+4}\right)^2 + \left(\frac{x^2}{x^2+4}\right)^3 + \dots$$

Sum =

$$f(x) = \frac{x^2(x^2+4)}{x^2+4}$$

$$(x^2+4) \left| - \frac{x^2(x^2+4)}{x^2+4} \right|$$

$$= \frac{x^2}{x^2+4-x^2}$$

$$f(x) = \frac{x^2}{4}$$

$$r = \frac{x^2}{x^2+4}$$

$$\left(-1 < \frac{x^2}{x^2+4} < 1\right) x^2+4$$

$$-x^2-4 < x^2 < x^2+4$$

$$\frac{-x^2-4 < x^2}{+x^2 \quad +x^2}$$

$$\frac{-4 < 2x^2}{2 \quad 2}$$

$$-2 < x^2$$

$$\frac{x^2 < x^2+4}{-x^2 \quad -x^2}$$
$$0 < 4$$

$$\text{I.O.C: } -\infty < x < \infty$$

Determine if the series converges or diverges using either the nth term test or recognizing if the series is geometric.

A)  $\sum_{n=1}^{\infty} \frac{5n+2}{3n-1}$  Diverges  $\lim_{n \rightarrow \infty} \frac{5n+2}{3n-1} = \frac{5}{3} \neq 0$

B)  $\sum_{n=1}^{\infty} \left(\frac{2}{5^n}\right)$   $\lim_{n \rightarrow \infty} \frac{2}{5^n} = 0$  Do another test

$\sum_{n=1}^{\infty} 2 \left(\frac{1}{5}\right)^n$  converges  $|r| = \frac{1}{5} < 1$

C)  $\sum_{n=1}^{\infty} \left(\frac{4^n}{50}\right)$  diverges  $\lim_{n \rightarrow \infty} \frac{4^n}{50} = \infty \neq 0$

$\sum_{n=1}^{\infty} \frac{1}{50} (4^n)$  diverges  $|r| = 4 > 1$

D)  $\sum_{n=1}^{\infty} \left(\frac{2n!}{5n!+3}\right)$   $\lim_{n \rightarrow \infty} \frac{2n!}{5n!+3} = \frac{2}{5} \neq 0$   
Diverges

What you'll Learn About  
 The Integral Test/P-Series Test/Comparison Test

Integral Test

converges  
 because the  
 area converges

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{1}{n(n+1)}$$

A)  $\sum_{n=1}^{\infty} \frac{1}{n^2} = 1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \frac{1}{25} + \dots$

$$\lim_{b \rightarrow \infty} \int_1^b \frac{1}{x^2} = \lim_{b \rightarrow \infty} \int_1^b x^{-2} = \lim_{b \rightarrow \infty} \left[ -x^{-1} \right]_1^b$$

$$= \lim_{b \rightarrow \infty} \left[ -\frac{1}{x} \right]_1^b = \lim_{b \rightarrow \infty} \left[ -\frac{1}{b} - (-1) \right] = 1$$

Area Converges

B)  $\sum_{n=1}^{\infty} \frac{1}{n}$

C)  $\sum_{n=1}^{\infty} \frac{1}{n^{1/3}}$