2015 AP Calculus BC Free Response

6. The Maclaurin Series for a function f is given by

$$\sum_{n=1}^{\infty} \frac{(-3)^{n-1}}{n} x^n = x - \frac{3}{2}x^2 + 3x^3 - \cdots + \frac{(-3)^{n-1}}{n}x^n + \cdots$$

and converges to f(x) for $|x| < R$, where R is the radius of convergence of the Maclaurin series.

a) Use the Ratio Test to find R.

$$\lim_{n\to\infty} \left| \frac{(-3)^n x^{n+1}}{(n+1)} \cdot \frac{n}{(-3)^{n-1} x^n} \right| = \left| \frac{x}{(-3)^{-1}} \right| = |3x| < 1$$

$$-\frac{1}{3} < \frac{3x}{3} < \frac{1}{3}$$

$$-\frac{1}{3} < x < \frac{1}{3}$$

$$R = \frac{1}{3}$$

20. What are all values of x for which the series $\sum_{n=1}^{\infty} \frac{(x-2)^n}{n3^n}$ converges?

(A) $-3 \cancel{\le} x \le 3$ (B) $-3 \cancel{<} x < 3$ (C) $-1 \cancel{<} x \le 5$ (D) $-1 \le x \le 5$ (E) $-1 \le x < 5$

$$\lim_{n\to\infty} \left| \frac{(x-2)^{n+1}}{(n+1)3^{n+1}} \cdot \frac{n3^n}{(x-2)^n} \right| = \left| \frac{x-2}{3} \right| < 1$$

$$-1 < \frac{x-2}{3} < 1$$

$$-3 < x-2 < 3$$

$$-1 < x < 5$$

$x = 5$

$\sum \frac{1}{n}$ Harmonic Diverges

$x = -1$

$$\sum_{n=1}^{\infty} \frac{(-3)^n}{n(3)^n}$$

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$$

Converges conditionally since this is the alternating harmonic