Right-Left	Find the area of the regions enclosed by the lines and curves 24. $x - y^2 = 0$ and $x + 2y^2 = 3$ $x = y^2$ $x = 3 - 2y^2$ $A = \int_{-1}^{1} (3 - 2y^2) - y^2$ $A = \int_{-1}^{1} 3 - 3y^2 = 3y - y^3 \Big]_{-1}^{1}$ $(3 - 1) - (-3 + 1)$ $2 - (-2) = 4$ $y^2 = 3 - 2y^2$ $+2y^2 \quad +2y^2$ $\frac{3y^2}{3} = \frac{3}{3}$ $y^2 = 1$ $y = \pm 1$
Top-Bottom	26. $4x^2 + y = 4$ and $x^4 - y = 1$ $y = 4 - 4x^2$ $-y = 1 - x^4$ $y = -1 + x^4$ $A = \int_{-1}^{1} (4 - 4x^2) - (-1 + x^4)$ $A = \int_{-1}^{1} 4 - 4x^2 + 1 - x^4$ $A = \int_{-1}^{1} -x^4 - 4x^2 + 5 = -\frac{1}{5}x^5 - \frac{4}{3}x^3 + 5x \Big]_{-1}^{1}$ $4 - 4x^2 = -1 + x^4$ $-4 + 4x^2 \quad -4 \quad +4x^2$ $0 = x^4 + 4x^2 - 5$ $0 = (x^2 + 5)(x^2 - 1)$ $x^2 - 1 = 0$ $x = \pm 1$ $\left(-\frac{1}{5} - \frac{4}{3} + 5\right) - \left(\frac{1}{5} + \frac{4}{3} - 5\right)$

$-\frac{2}{5} - \frac{8}{3} + 10$

	Find the area of the regions enclosed by the lines and curves 18. $y = x^4 - 4x^2 + 4$ and $y = x^2$ $x^4 - 4x^2 + 4 = x^2$ $-x^2 \quad -x^2$ $x^4 - 5x^2 + 4 = 0$ $(x^2 - 4)(x^2 - 1)$ $x = \pm 2 \quad x = \pm 1$
Right - Left Solve for x	23. $y^2 - 4x = 4$ and $4x - y = 16$ $\frac{-4x}{-4} = \frac{4}{-4} - \frac{y^2}{-4}$ $\quad$ $\frac{4x}{4} = \frac{16}{4} + \frac{y}{4}$ $x = -1 + \frac{1}{4}y^2$ $\quad$ $x = 4 + \frac{1}{4}y$ $A = \int_{-4}^{5} (4 + \frac{1}{4}y) - (-1 + \frac{1}{4}y^2)$ $\boxed{30.375}$ $4(-1 + \frac{1}{4}y^2 = 4 + \frac{1}{4}y)$ $-4 + y^2 = 16 + y$ $-y \quad -16 \quad -16 \quad -y$ $y^2 - y - 20 = 0$ $(y-5)(y+4) = 0$

$y = 5 \quad y = -4$

Page 18

What you'll Learn About

- Finding lengths of curves

$L = \int \sqrt{1 + \left(\frac{dy}{dx}\right)^2}$

2. Use your calculator to find the length of the curve

$y = \tan x \quad \frac{-\pi}{3} \le x \le 0$

$\frac{dy}{dx} = \sec^2 x$

$L = \int_{-\frac{\pi}{3}}^{0} \sqrt{1 + (\sec^2 x)^2}\, dx$

$L = \int \sqrt{1 + \left(\frac{dx}{dy}\right)^2}$

4. Use your calculator to find the length of the curve

$x = \sqrt{1 - y^2} \quad \frac{-1}{2} \le y \le \frac{1}{2}$

$x = (1 - y^2)^{1/2}$

$\frac{dx}{dy} = \frac{1}{2}(1 - y^2)^{-1/2} \cdot (-2y) = \frac{-y}{\sqrt{1 - y^2}}$

$L = \int_{-\frac{1}{2}}^{\frac{1}{2}} \sqrt{1 + \left(\frac{-y}{\sqrt{1 - y^2}}\right)^2}$

8. Use your calculator to find the length of the curve

$x = \int_0^{y^2} \sqrt{\sec^2 t - 1} \quad \frac{-\pi}{3} \le y \le \frac{\pi}{4}$

$L = \int_{-\frac{\pi}{3}}^{\pi/4} \sqrt{1 + (\quad)^2}$

$\frac{dx}{dy} = \sqrt{\sec^2(y^2) - 1} \cdot 2y$

$\left(\frac{dx}{dy}\right)^2 = 4y^2(\sec^2(y^2) - 1)$

$L = \int_{-\frac{\pi}{3}}^{\pi/4} \sqrt{1 + (4y^2(\sec^2(y^2) - 1)}$

Direction

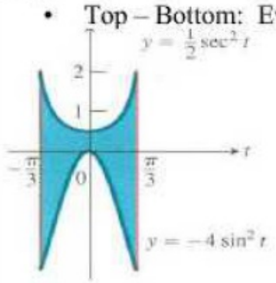
- A particle is stopped when the velocity = 0
- A particle moves left when the velocity is negative
- A particle moves right when the velocity is positive

Displacement/Total Distance

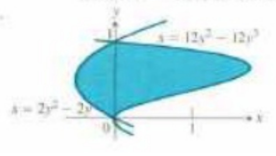
- Displacement is the integral of the velocity
- Total Distance is the integral of the absolute value of the velocity
    - Remember when doing total distance by hand you must find when the particle is moving left and right and split up your integral doing the absolute value of the part that is moving left

Area

- Top – Bottom: Everything in the integral is in terms of x



- Right – Left: Everything in the integral is in terms of y.



Arc Length

$$L = \int \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx \text{ if original equation is solved for y}$$

$$L = \int \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy \text{ if original equation is solved for x}$$