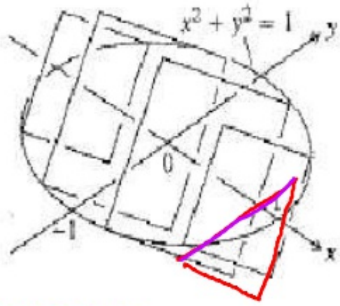
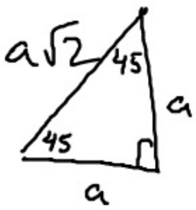
Find the volume of the solid which lies between planes perpendicular to the x-axis at x = -1 and x =1 between the semi-circles $y=-\sqrt{1-x^2}$ and $y=\sqrt{1-x^2}$. The cross sections perpendicular to the x-axis are squares with diagonals running from $y=-\sqrt{1-x^2}$ to $y=\sqrt{1-x^2}$.



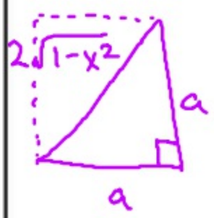
Top – Bottom

$\sqrt{1-x^2}-(-\sqrt{1-x^2})$

Diagonal of Square $=2\sqrt{1-x^2}$



$$V=\int_{-1}^{1}(\text{Area of Square})\,dx$$



$$a^2+a^2=\left(2\sqrt{1-x^2}\right)^2$$

$$\frac{2a^2}{2}=\frac{4(1-x^2)}{2}$$

$$a^2=2(1-x^2)=2-2x^2$$

$$a=\sqrt{2(1-x^2)}$$

$$a=\sqrt{2}\sqrt{1-x^2}$$

$$V=2\int_{-1}^{1}(1-x^2)\,dx$$

$$V=2\left[x-\frac{1}{3}x^3\right]_{-1}^{1}$$

$$V=2\left[\frac{2}{3}-\left(-\frac{2}{3}\right)\right]$$

$$\boxed{V=\frac{8}{3}}$$

The solid lies between planes perpendicular to the x-axis at x = 0 and x =4. the cross sections perpendicular to the x-axis between these planes run from $y=-\sqrt{x}$ and $y=\sqrt{x}$. If the cross-sections are equilateral triangles with one side running from $y=-\sqrt{x}$ and $y=\sqrt{x}$

Top – Bottom

$\sqrt{x}-(-\sqrt{x})$

base = side of eq. $\Delta = 2\sqrt{x}$

$$h^2+(\sqrt{x})^2=(2\sqrt{x})^2$$

$$h^2+x=4x$$

$$h^2=3x$$

$$h=\sqrt{3x}=\sqrt{3}\sqrt{x}$$

$$V=\int_0^4(\text{Area of }\Delta)\,dx$$

$$V=\int_0^4\frac{1}{2}bh\,dx$$

$$V=\frac{1}{2}\int_0^4 2\sqrt{x}\sqrt{3}\sqrt{x}\,dx$$

$$V=\sqrt{3}\int_0^4 x\,dx=\sqrt{3}\left[\frac{1}{2}x^2\right]_0^4$$

$$=8\sqrt{3}$$