What you'll Learn About

- Analyzing antiderivatives graphically
- Connecting Antiderivatives to Area
- Taking the derivative of an integral

A) Find $\frac{d}{dx}\int_1^x (\cos t)dt$

$\frac{d}{dx}\left[\sin t\right]_1^x$

$\frac{d}{dx}\left[\sin x - \sin(1)\right]$

$\boxed{\frac{d}{dx}\int_1^x \cos t\,dt = \cos x}$

B) Find $\frac{d}{dx}\int_1^{x^3} (\cos t)dt$

$\frac{d}{dx}\left[\sin t\right]_1^{x^3}$

$\frac{d}{dx}\left[\sin(x^3) - \sin 1\right]$

$\boxed{\frac{d}{dx}\int_1^{x^3} \cos t\,dt = 3x^2\cos(x^3)}$

C) Find $\frac{d}{dx}\int_{x^3}^{x^2} (\cos t)dt$

$\frac{d}{dx}\left[\sin t\right]_{x^3}^{x^2}$

$\frac{d}{dx}\left[\sin(x^2) - \sin x^3\right]$

$\boxed{\frac{d}{dx}\int_{x^3}^{x^2} \cos t\,dt = 2x\cos(x^2) - 3x^2\cos(x^3)}$

$\frac{d}{dx}$ lower limit

Find $\frac{dy}{dx}$ for the given function

$\frac{d}{dx}$ upper limit

2) $y=\int_2^x \left(3t+\cos t^2\right)dt$

$\frac{dy}{dx} = \left(3x+\cos x^2\right)\cdot 1 - \left(6+\cos 4\right)\cdot 0$

$\boxed{\frac{dy}{dx} = 3x+\cos x^2}$

10) $y=\int_6^{x^2} \left(\cot(3t)\right)dt$

$\boxed{\frac{dy}{dx} = \cot\left(3x^2\right)\cdot 2x}$

12) $y=\int_{\pi}^{\pi-x} \left(\frac{1+\sin^2 t}{1+\cos^2 t}\right)dt$

$\frac{dy}{dx} = \left(\frac{1+\sin^2(\pi-x)}{1+\cos^2(\pi-x)}\right)(-1)$

14) $y=\int_x^7 \left(\sqrt{2t^4+t+1}\right)dt$

$y = -\int_7^x \sqrt{2t^4+t+1}\,dt$

$\boxed{\frac{dy}{dx} = -\sqrt{2x^4+x+1}}$

20) $y=\int_{\sin x}^{\cos x} \left(t^2\right)dt$

$\frac{d}{dx}(\cos x)$ $\qquad$ $\frac{d}{dx}(\sin x)$

$\frac{dy}{dx} = (\cos x)^2\cdot(-\sin x) - (\sin x)^2(\cos x)$

$g=\int_{-2}^{-5} f(t)\,dt = 2$ $(-5,2)$ $g=\int_{-2}^{-2} f(t)\,dt = 0$ $(-2,0)$ $g=\int_{-2}^{1} f(t)\,dt = 2$ $(1,2)$ $g=\int_{-2}^{4} f(t)\,dt = 0$ $(4,0)$ $g=\int_{-2}^{5} f(t)\,dt = \frac{1}{2}$ $(5,\frac{1}{2})$

Using the Fundamental Theorem of Calculus

$g(x)=\int_{-2}^{x} f(t)\,dt$

$\boxed{g'(x)=f(x)}$

function value (y-coordinate of given graph)

$g''(x)=f'(x)$

Slope of given graph

Area between f(t) and the x-axis from -2 to x

Graph of $f(t)$

Given: $g(x)=\int_{-2}^{x} f(t)\,dt$. Find each of the following:

1. $g(4) = g(4) = \int_{-2}^{4} f(t)\,dt = 0$

2. $g'(1) = 0$

3. $g''(-1) =$ undefined

4. $g''(-3) =$ undefined

5. $g'(0) = 1$

6. $g(1) = \int_{-2}^{1} f(t)\,dt = 2$

7. $g(-3) = \int_{-2}^{-3} f(t)\,dt = \frac{1}{2}$

$= -\int_{-3}^{-2} f(t)\,dt$

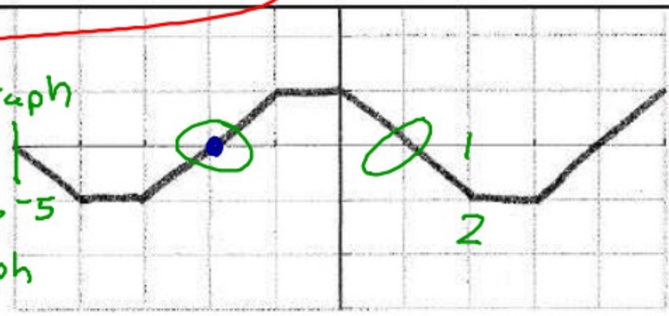
8. $g(-4) = \int_{-2}^{-4} f(t)\,dt = \frac{3}{2}$

$g(x) = \int_{-2}^{x} f(t)\,dt \rightarrow$ Area

g(x) → original function
→ find area to find g's coordinates

$g'(x) = f(x) \rightarrow$ Graph

$g'' = f'(x) \rightarrow$ Slope of Graph



g'(x) = given graph

Graph of $f(t)$

9. Find the equation of the tangent line to the graph of g at x = -2

Point $g(-2) = \int_{-2}^{-2} f(t)\,dt = 0$ $(-2, 0)$

Slope $g'(-2) = 0$ $m = 0$

$y = 0 + 0(x+2) = 0$ $y = 0$

10. Determine any relative/local maxima or minima on the interval (-5,2)

C.P
$x = -2$, $x = 1$

Local min b/c sign of g' changes from − to +

Local max b/c sign of g' changes from + to −

Local max/min

C.P. $g'(x) = 0 \leftarrow$ Find the x-int of given graph

Intervals of Inc/Dec $\leftarrow$ $g'(-3) = -1 < 0$ dec $\{ g'(0) = 1 > 0$ inc $\{ g'(2) = -1 < 0$ dec

OR

2nd derivative test $\leftarrow$ $g''(-2) = 1 > 0$ $g''(1) = -1 < 0$

11. Determine the absolute maximum and minimum of g on [-5, 2].

Absolutes

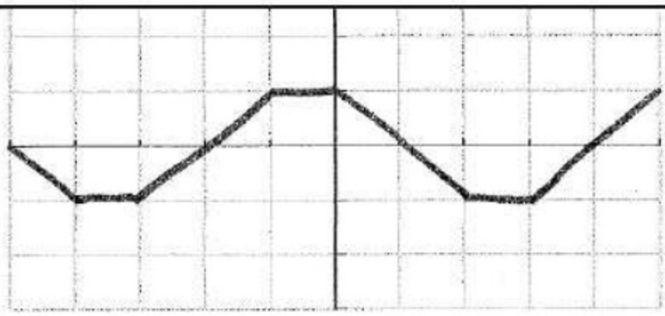
Plug endpts and critical points into original

$g(-5) = \int_{-2}^{-5} f(t)\,dt = 2$ max

$g(-2) = \int_{-2}^{-2} f(t)\,dt = 0$ min

$g(1) = \int_{-2}^{1} f(t)\,dt = 2$ max

$g(2) = \int_{-2}^{2} f(t) = 1.5$



Graph of $f(t)$

11. Let $h(x) = g(x) - .5x^2 - x$. Determine the critical values of $h(x)$ on $-5 < x < 5$.

12. Let $n(x) = [g(x)]^2 + f(x)$. Find $n'(1) =$