

Find the equation of the tangent line

F)  $y = \csc^{-1}x$  at  $x=2$

$$\frac{dy}{dx} = \frac{-1}{|x|\sqrt{x^2-1}}$$

$$\left. \frac{dy}{dx} \right|_{x=2} = \frac{-1}{|2|\sqrt{3}} = -\frac{1}{2\sqrt{3}}$$

ratio  $\downarrow$  angle  $\downarrow$

$$y = \csc^{-1}(2)$$

$$y = \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$$

$$y = \frac{\pi}{6} - \frac{1}{2\sqrt{3}}(x-2)$$

$$y = \csc x \rightarrow y = \frac{1}{\sin x}$$

G) Find the derivative of  $f(x) = \sin x$  at  $x = \frac{\pi}{6}$

$$f'(x) = \cos x$$

$$f'\left(\frac{\pi}{6}\right) = \cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}$$

H) Find the derivative of  $f(x) = \arcsin x$  at  $x = \frac{1}{2}$

$$f'(x) = \frac{1}{\sqrt{1-x^2}}$$

$$f'\left(\frac{1}{2}\right) = \frac{1}{\sqrt{1-\left(\frac{1}{2}\right)^2}}$$

$g(x)$  is inverse  
of  $f(x)$

$f(x)$	$g(x)$
$(3, 15)$	$(15, 3)$
$m = -8$	$m = -\frac{1}{8}$

$f(x)$	$f^{-1}(x)$
$(8, 4)$	$(4, 8)$
$m = 3$	$m = \frac{1}{3}$

1. Let  $f$  be a differentiable function such that  $f(3) = 15$ ,  $f(6) = 3$ ,  $f'(3) = -8$  and  $f'(6) = -2$ .

The function  $g$  is differentiable and  $g(x) = f^{-1}(x)$  for all  $x$ . What is the value of  $g'(15)$ ?

- a)  $-1/2$
- b)  $-1/8$
- c)  $1/6$
- d)  $1/3$
- e) The value of  $g'(15)$  cannot be determined

2. Let  $f$  be a differentiable function such that  $f(3) = 5$ ,  $f(8) = 4$ ,  $f'(3) = 6$  and  $f'(8) = 3$ .

The function  $g$  is differentiable and  $g(x) = f^{-1}(x)$  for all  $x$ . What is the value of  $g'(4)$ ?

- a)  $-1/2$
- b)  $-1/8$
- c)  $1/6$
- d)  $1/3$
- e) The value of  $g'(4)$  cannot be determined

3. Let  $f$  be a differentiable function such that  $f(3) = 5$ ,  $f(8) = 4$ ,  $f'(3) = 6$  and  $f'(8) = 3$ .

The function  $g$  is differentiable and  $g(x) = f^{-1}(x)$  for all  $x$ . What is the value of  $g'(5)$ ?

- a)  $-1/2$    b)  $-1/8$    c)  $1/6$    d)  $1/3$   
 e) The value of  $g'(5)$  cannot be determined

4. If  $f(2) = -3$ ,  $f'(2) = \frac{4}{3}$ , and  $g(x) = f^{-1}(x)$ ,

$$y = 2 + \frac{3}{4}(x+3)$$

what is the equation of the tangent line to  $g(x)$  at  $x = -3$ ?

- A)  $y-2 = \frac{-3}{4}(x+3)$       B)  $y+2 = \frac{-3}{4}(x-3)$   
 C)  $y-2 = \frac{3}{4}(x+3)$       D)  $y+3 = \frac{3}{4}(x-2)$   
 E)  $y-2 = \frac{4}{3}(x+3)$

5. If  $f(2) = -3$ ,  $f'(2) = \frac{-4}{3}$ , and  $g(x) = f^{-1}(x)$ ,

what is the equation of the tangent line to  $g(x)$  at  $x = -3$ ?

- A)  $y-2 = \frac{-3}{4}(x+3)$       B)  $y+2 = \frac{-3}{4}(x-3)$   
 C)  $y-2 = \frac{3}{4}(x+3)$       D)  $y+2 = \frac{4}{3}(x-3)$   
 E)  $y-2 = \frac{4}{3}(x+3)$

$f(x)$	$g(x) = f^{-1}(x)$
$(2, -3)$	$(-3, 2)$
$m = \frac{4}{3}$	$m = \frac{3}{4}$

$f(2) = -3$     $f'(2) = -\frac{4}{3}$   
 $(2, -3)$     $m = -\frac{4}{3}$

$g \rightarrow f^{-1}(x)$   
 $(-3, 2)$     $m = \frac{3}{4}$

6. If  $f(2) = -3$ ,  $f'(2) = \frac{-3}{4}$ , and  $g(x) = f^{-1}(x)$ ,

what is the equation of the tangent line to  $g(x)$   
at  $x = -3$ ?

~~A~~  $y - 2 = \frac{-3}{4}(x + 3)$

B)  $y + 3 = \frac{-4}{3}(x + 2)$

~~C~~  $y - 2 = \frac{3}{4}(x + 3)$

~~D~~  $y + 2 = \frac{4}{3}(x - 3)$

E)  $y - 2 = \frac{-4}{3}(x + 3)$

What you'll Learn About  
 How to take the derivative of exponential and logarithmic functions

~~$y = 5^x$   
 $y' = x \cdot 5^{x-1}$~~

$y = x^5$   
 $y' = 5x^4$

$y = \ln(5e)$   
 $y = \ln 5 + \ln e$   
 $y = \ln 5 + 1$

$y = \ln_e(5e)$

$e^y = 5e$

$y = \ln_e 5$

$e^y = e$

A)  $y = 5^x$   
 $y' = 5^x \cdot \ln 5 \cdot 1$

B)  $y = 7^{x^2}$   
 $y' = 7^x \cdot \ln 7 \cdot 2x$

C)  $y = 5^{\sin x}$   
 $y' = 5^{\sin x} \cdot \ln 5 \cdot \cos x$

D)  $y = 6^{\arctan x^3}$   $y = 6$

$y' = 6^{\arctan x^3} \cdot \ln 6 \cdot \frac{1}{1+(x^3)^2} \cdot 3x^2$

E)  $y = e^x$   
 $y' = e^x \cdot (\ln e) \cdot 1$

F)  $y = 5e^{5x}$   
 $y' = 5(e^{5x}) \ln e \cdot 5$

$y' = e^x$

$y' = 25e^{5x}$

G)  $y = (5e)^{5x}$   
 $y' = (5e)^{5x} \cdot \ln(5e) \cdot 5$

H)  $y = e^{-\frac{3}{4}x}$   
 $y' = e^{-\frac{3}{4}x} \cdot \ln e \cdot -\frac{3}{4}$

I)  $y = x^3 e^{4x} - x^4 e^{2x}$   
 $y' = x^3(e^{4x} \cdot \ln e \cdot 4) + e^{4x}(3x^2) - [x^4(e^{2x} \cdot \ln e \cdot 2) + e^{2x}(4x^3)]$

$y' = 4x^3 e^{4x} + 3x^2 e^{4x} - 2x^4 e^{2x} - 4x^3 e^{2x}$