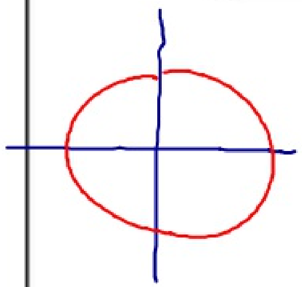
What you'll Learn About
How to take the derivative of a function that is not solved for y (an implicitly defined function)



$y = (x)^2$

$\frac{dy}{dx} = 2(x)^1 \cdot \frac{dx}{dx}$

$= 2x \cdot 1$

$= 2x$

$\frac{d}{dx}(y)^2$

$2(y)^1 \cdot \frac{dy}{dx}$

Find the derivative of the following function

A) $x^2 + y^2 = 1$

$-x^2 \quad -x^2$

$y^2 = 1 - x^2$

$y = \pm\sqrt{1-x^2} = \pm(1-x^2)^{1/2}$

$\frac{dy}{dx} = \pm\frac{1}{2}(1-x^2)^{-1/2} \cdot (-2x) = \boxed{\frac{\pm(-x)}{\sqrt{1-x^2}}}$

B) $x = \cos\theta \quad y = \sin\theta$

$\boxed{\frac{dy}{dx} = \frac{\cos\theta}{-\sin\theta}}$

C) $x^2 + y^2 = 1$

$2x + 2y\frac{dy}{dx} = 0$

$-2x \qquad -2x$

$\frac{2y\frac{dy}{dx}}{2y} = \frac{-2x}{2y}$

$\boxed{\frac{dy}{dx} = \frac{-x}{y}}$

$\frac{dy}{dx} = \frac{-x}{\pm\sqrt{1-x^2}}$

D) $x^2 + y^2 = xy$

$\cancel{2x} + 2y\frac{dy}{dx} = x\frac{dy}{dx} + y(1)$

$\cancel{-2x} \quad -x\frac{dy}{dx} \qquad -x\frac{dy}{dx} \qquad -2x$

$2y\frac{dy}{dx} - x\frac{dy}{dx} = y - 2x$

$\frac{\frac{dy}{dx}(2y - x)}{2y - x} = \frac{y - 2x}{2y - x}$

$\boxed{\frac{dy}{dx} = \frac{y-2x}{2y-x}}$

E) $x^2 = \frac{x-y}{x+y}$

$$(x+y)^2\, 2x = \frac{(x+y)\left(1 - \frac{dy}{dx}\right) - (x-y)\left(1 + \frac{dy}{dx}\right)}{\cancel{(x+y)^2}}$$

$$2x(x+y)^2 = \cancel{x} - x\frac{dy}{dx} + y - \cancel{y\frac{dy}{dx}} - \cancel{x} - x\frac{dy}{dx} + y + \cancel{y\frac{dy}{dx}}$$

$$\frac{2x(x+y)^2}{2} = \frac{2y}{2} - \frac{2x\frac{dy}{dx}}{2}$$

$$x(x+y)^2 = y - x\frac{dy}{dx}$$

$$-y \quad -y$$

$$\frac{x(x+y)^2 - y}{-x} = \frac{-x\frac{dy}{dx}}{-x}$$

~~F) $x + \tan(xy) = y$~~

$$\boxed{\frac{x(x+y)^2 - y}{-x}} \leftarrow \boxed{-(x+y)^2 + \frac{y}{x} = \frac{dy}{dx}}$$

F

$$x + \tan(xy) = y$$

$$1 + \sec^2(xy)\cdot\left[x\frac{dy}{dx} + y(1)\right] = \frac{dy}{dx}$$

$$1 + x\sec^2(xy)\frac{dy}{dx} + y\sec^2(xy) = \frac{dy}{dx}$$

$$-x\sec^2(xy)\frac{dy}{dx} \qquad -x\sec^2(xy)\frac{dy}{dx}$$

$$1 + y\sec^2(xy) = \frac{dy}{dx} - x\sec^2(xy)\frac{dy}{dx}$$

$$\frac{1 + y\sec^2(xy)}{1 - x\sec^2(xy)} = \frac{\frac{dy}{dx}\left(1 - x\sec^2(xy)\right)}{\left(1 - x\sec^2(xy)\right)}$$

$$\boxed{\frac{1 + y\sec^2(xy)}{1 - x\sec^2(xy)} = \frac{dy}{dx}}$$