Find the equation for the tangent line at the given point

Q) \( y = \frac{x^5 + 2x}{x^2} \) at \( x = 1 \)

R) \( y = 5x^2 + 3 \) at \( x = 3 \)

S) Find an equation of the line perpendicular to the tangent to the curve \( y = 4x^3 - 6x + 2 \) at the point \( (2, 22) \).

T) Find the points on the curve \( y = x^3 - 3x^2 - 9 \) where the tangent is parallel to the \( x \)-axis.
U) Suppose $u$ and $v$ are differentiable functions at $x = 2$ and $u(2) = 3$, $u'(2) = 3$, $v(2) = 1$, $v'(2) = 2$

i) Find $\frac{d}{dx}(uv)$

ii) Find $\frac{d}{dx}(\frac{u}{v})$

iii) Find $\frac{d}{dx}(3u - 2v + 2uv)$

V) Find the derivative of $y = x$ with respect to $x$

W) Find the derivative of $y = x$ with respect to $t$

X) Find the derivative of $y = x$ with respect to $P$
Flowchart: Selecting a Procedure for Derivatives

Step 1
Categorize the function.

- Is it a quotient? No
- Is it a product? No
- Is it a function within a function? No
- Is it a basic function? Yes

Can you change this function into another form? No

Revise the form to something easier.

Step 2
Differentiate using the rule that would apply in Step 1.
Find equations for the lines that are tangent and normal to the graph of \( y = 2\cos x \) at \( x = \frac{\pi}{4} \).

Find the points on the curve \( y = \cot x \), \( 0 \leq x \leq \frac{\pi}{2} \), where the tangent line is parallel to the line \( y = -2x \).
1. Let \( f \) be a differentiable function such that 
\[ f(3) = 15, \quad f(6) = 3, \quad f'(3) = -8 \text{ and} \]
\[ f'(6) = -2. \]
The function \( g \) is differentiable and 
\[ g(x) = f^{-1}(x) \text{ for all } x. \]
What is the value of \( g'(15) \)?

a) -1/2    b) -1/8    c) 1/6    d) 1/3    e) The value of \( g'(15) \) cannot be determined

2. Let \( f \) be a differentiable function such that 
\[ f(3) = 5, \quad f(8) = 4, \quad f'(3) = 6 \text{ and} \]
\[ f'(8) = 3. \]
The function \( g \) is differentiable and 
\[ g(x) = f^{-1}(x) \text{ for all } x. \]
What is the value of \( g'(4) \)?

a) -1/2    b) -1/8    c) 1/6    d) 1/3    e) The value of \( g'(4) \) cannot be determined
3. Let \( f \) be a differentiable function such that \( f(3) = 5, f(8) = 4, f'(3) = 6 \) and \( f'(8) = 3 \).

The function \( g \) is differentiable and \( g(x) = f^{-1}(x) \) for all \( x \). What is the value of \( g'(5) \)?

a) -1/2  b) -1/8  c) 1/6  d) 1/3  e) The value of \( g'(5) \) cannot be determined

4. If \( f(2) = -3, f'(2) = \frac{4}{3} \), and \( g(x) = f^{-1}(x) \),

what is the equation of the tangent line to \( g(x) \) at \( x = -3 \)?

A) \( y-2 = -\frac{3}{4}(x + 3) \)  B) \( y+2 = -\frac{3}{4}(x - 3) \)

C) \( y-2 = \frac{3}{4}(x + 3) \)  D) \( y+3 = \frac{3}{4}(x - 2) \)

E) \( y-2 = \frac{4}{3}(x + 3) \)
5. If \( f(2) = -3, \) \( f'(2) = -\frac{4}{3}, \) and \( g(x) = f^{-1}(x), \)
what is the equation of the tangent line to \( g(x) \) at \( x = -3? \)

A) \( y - 2 = -\frac{3}{4}(x + 3) \) 

B) \( y + 2 = -\frac{3}{4}(x - 3) \)

C) \( y - 2 = \frac{3}{4}(x + 3) \) 

D) \( y + 2 = \frac{4}{3}(x - 3) \)

E) \( y - 2 = \frac{4}{3}(x + 3) \)

6. If \( f(2) = -3, \) \( f'(2) = -\frac{3}{4}, \) and \( g(x) = f^{-1}(x), \)
what is the equation of the tangent line to \( g(x) \) at \( x = -3? \)

A) \( y - 2 = -\frac{3}{4}(x + 3) \) 

B) \( y + 3 = -\frac{4}{3}(x + 2) \)

C) \( y - 2 = \frac{3}{4}(x + 3) \) 

D) \( y + 2 = \frac{4}{3}(x - 3) \)

E) \( y - 2 = -\frac{4}{3}(x + 3) \)
Graph the parametric function given

A) \[ x = t^2 - 3 \quad y = t \quad t \geq 0 \]

B) Find the derivative of the function at \( t = 5 \)

C) Find the equation of the tangent line at \( t = 1 \)

\[ x = 3t \quad y = 9t^2 \]

D) Find the equation of the tangent line at \( \theta = \frac{\pi}{4} \)

\[ x = \cos \theta \quad y = \sin \theta \]

E) Find the equation of the tangent line at \( t = \pi \)

\[ x = \sec^2(2t) - 1 \quad y = \tan(2t) \]
A curve C is defined by the parametric equations \( x = t^2 - 4t + 1 \) and \( y = t^3 \). Determine the equation of the line tangent to the graph of C at the point (1, 64)?

Determine the horizontal and vertical tangents for the parametric curve

\[ \begin{align*}
A) & \quad x = 1 - t \quad y = t^2 - 4t \\
B) & \quad x = \cos \theta \quad y = 2\sin(2\theta)
\end{align*} \]
The derivative of \( e^x \) is: \( (\text{Itself})(\text{Derivative of the power}) \)

\[
\frac{d}{dx} e^u = e^u \frac{du}{dx}
\]

The derivative of \( a^x \) is:
\( (\text{Itself})(\ln \text{ of the base})(\text{Derivative of the power}) \)

\[
\frac{d}{dx} (a^u) = a^u \ln a \frac{du}{dx}
\]

The derivative of \( a^x \) is:
\( (\text{Itself})(\ln \text{ of the base})(\text{Derivative of the power}) \)

\[
\frac{d}{dx} a^u = a^u \ln a \frac{du}{du}
\]

The derivative of \( \ln u \) is:
\( (\text{one over what you are taking the } \ln \text{ of}) \text{ times now you should be in the numerator (Derivative of what you are taking the } \ln \text{ of}) \)

\[
\frac{d}{dx} \ln u = \frac{1}{u} \frac{du}{dx}
\]

- One over the square root of 1 – the ratio squared all times the derivative of the ratio.

\[
\frac{d}{dx} \sin^{-1} u = \frac{1}{\sqrt{1-u^2}} \frac{du}{dx}
\]

- Negative One over the square root of 1 – the ratio squared all times the derivative of the ratio.

\[
\frac{d}{dx} \cos^{-1} u = -\frac{1}{\sqrt{1-u^2}} \frac{du}{dx}
\]
\[
\frac{d}{dx} \tan^{-1} u = \frac{1}{1+u^2} \frac{du}{dx}
\]

- One over 1 + the ratio squared all times the derivative of the ratio.

\[
\frac{d}{dx} \cot^{-1} u = -\frac{1}{1+u^2} \frac{du}{dx}
\]

- Negative One over 1 + the ratio squared all times the derivative of the ratio.

\[
\frac{d}{dx} \sec^{-1} u = \frac{1}{|u|\sqrt{u^2-1}} \frac{du}{dx}
\]

- One over the absolute value of the ratio times the square root of the ratio squared minus 1 all times the derivative of the ratio.

\[
\frac{d}{dx} \csc^{-1} u = -\frac{1}{|u|\sqrt{u^2-1}} \frac{du}{dx}
\]

Negative One over the absolute value of the ratio times the square root of the ratio squared minus 1 all times the derivative of the ratio.
When you do the power rule the base does not change only the power
- Once you have done the power rule, you are done with the powers

When you do the derivative of a trig function the angle does not change

**Chain Rule**

- **Polynomial**
  - (Power Rule)(Derivative Base)
    \[ y = (1 + x^2)^5 \]
    \[ y' = 5(1 + x^2)^4 \cdot 2x \]

- **Trig Function**
  - (Power rule)(Derivative of base)(Derivative of angle)
    \[ y = \sin^5 (3x) \]
    \[ y' = 5 \sin^4 (3x) \cdot (\cos \beta x) \cdot 3 \]
Chain Rule

- Product and quotient rule over rule everything when you have 2 functions

\[ y = x (\sin 3x)^{1/2} \]

\[ y' = x \left[ \frac{1}{2} (\sin 3x)^{-1/2} \cdot (\cos(3x)) \cdot 3 \right] + (\sin 3x) \]

- If the base is a product or quotient rule then you must start with the power rule

\[ y = (x \sin 3x)^{1/2} \]

\[ y' = \frac{1}{2} (x \sin 3x)^{-1/2} \cdot [x(\cos(3x)) \cdot 3] + (\sin 3x) \]