

Point - Slope

$$y - y_1 = m(x - x_1) \rightarrow y = y_1 + m(x - x_1)$$

Tangent Line

* Find slope at the given point

Take derivative

* Find the point of tangency

Plug x into the original eq.

Find the equation for the tangent line at the given point

Q) $y = \frac{x^5 + 2x}{x^2}$ at $x = 1$

x_1, y_1
 $(1, 3)$

$$y = \frac{x^5 + 2x}{x^2}$$

$$y = 3 + 1(x - 1)$$

$$\frac{dy}{dx} = \frac{x^2(5x^4 + 2) - (x^5 + 2x)(2x)}{(x^2)^2}$$

$$\left. \frac{dy}{dx} \right|_{x=1} = \frac{(5+2) - (1+2)(2)}{1} = 1$$

$$\left. \frac{dy}{dx} \right|_{x=1} = 1 \leftarrow m$$

R) $y = 5x^2 + 3$ at $x = 3$

$$y = 5x^2 + 3 \quad x = 3$$

(x_1, y_1)
 $(3, 48)$

$$\frac{dy}{dx} = 10x$$

$$\left. \frac{dy}{dx} \right|_{x=3} = 10(3) = 30$$

$$y = 48 + 30(x - 3)$$

S) Find an equation of the line perpendicular to the tangent to the curve $y = 4x^3 - 6x + 2$ at the point $(2, 22)$.

$$\frac{dy}{dx} = 12x^2 - 6$$

x_1, y_1

(normal)
slope opposite reciprocal

$$\left. \frac{dy}{dx} \right|_{x=2} = 12(2)^2 - 6 = 42$$

$$y = 22 - \frac{1}{42}(x - 2)$$

Horizontal Tangents (slope = 0)

T) Find the points on the curve $y = x^3 - 3x^2 - 9$ where the tangent is parallel to the x-axis

$$y = x^3 - 3x^2 - 9$$

$$x = 0$$

$$(0, -9)$$

$$x = 2$$

$$(2, -13)$$

$$\frac{dy}{dx} = 3x^2 - 6x$$

$$0 = 3x^2 - 6x$$

$$0 = 3x(x - 2)$$

$$2^3 - 3(2)^2 - 9$$

$$8 - 12 - 9$$

U) Suppose u and v are differentiable functions at $x = 2$ and

$$u(2) = 3, u'(2) = 3, v(2) = 1, v'(2) = 2$$

i) Find $\frac{d}{dx}(uv) = u \left(\frac{dv}{dx} \right) + v \left(\frac{du}{dx} \right) = u \cdot v' + v(u')$

$$\frac{d}{dx}(uv) = 9$$

$$\begin{array}{c} \downarrow \\ u(2) \cdot v'(2) + v(2)u'(2) \\ \downarrow \quad \downarrow + (1)(3) \\ (3)(2) + (1)(3) \\ 9 \end{array}$$

ii) Find $\frac{d}{dx} \left(\frac{u}{v} \right) = \frac{v(u') - u(v')}{v^2} = \frac{(1)(3) - 3(2)}{1^2}$

iii) Find $\frac{d}{dx}(3u - 2v + 2uv)$

V) Find the derivative of $y = x$ with respect to x

W) Find the derivative of $y = x$ with respect to t

X) Find the derivative of $y = x$ with respect to P