Tangent Line

* Find slope at the given point
  - Take derivative
  - Plug x into the original eq.

Horizontal Tangents (Slope = 0)

Find the equation for the tangent line at the given point

Q) \( y = \frac{x^3 + 2x}{x^2} \) at \( x = 1 \)

\[ x_1, y_1 \quad (1, 3) \]

\[ y = \frac{x^5 + 2x}{x^2} \]

\[ \frac{dy}{dx} = \frac{x^2(5x^2 + 2) - (x^5 + 2x)(2x)}{(x^2)^2} \]

\[ \frac{dy}{dx} \bigg|_{x=1} = \frac{(5+2)-(1+2)(2)}{1} = 1 \]

\[ \frac{dy}{dx} \bigg|_{x=1} = 1 \Longleftrightarrow m \]

\[ y = 3 + 1(x-1) \]

R) \( y = 5x^2 + 3 \) at \( x = 3 \)

\[ y = 5x^2 + 3 \]

\[ x = 3 \]

\[ y, y_1 \quad (3, 48) \]

\[ \frac{dy}{dx} = 10x \]

\[ \frac{dy}{dx} \bigg|_{x=3} = 10(3) = 30 \]

\[ y = 48 + 30(x-3) \]

S) Find an equation of the line perpendicular to the tangent to the curve \( y = 4x^3 - 6x + 2 \) at the point \( (2, 22) \).

\[ \frac{dy}{dx} = 12x^2 - 6 \]

\[ \frac{dy}{dx} \bigg|_{x=2} = 12(2)^2 - 6 = 42 \]

(normal) slope opposite reciprocal

\[ y = 22 - \frac{1}{42} (x-2) \]

<table>
<thead>
<tr>
<th>x</th>
<th>y = 22 - \frac{1}{42} (x-2)</th>
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</thead>
<tbody>
<tr>
<td>0</td>
<td>( (0, -9) )</td>
</tr>
<tr>
<td>2</td>
<td>( (2, -13) )</td>
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T) Find the points on the curve \( y = x^3 - 3x^2 - 9 \) where the tangent is parallel to the \( x \)-axis.

\[ y = x^3 - 3x^2 - 9 \]

\[ \frac{dy}{dx} = 3x^2 - 6x \]

\[ 0 = 3x^2 - 6x \]

\[ 0 = 3x(x-2) \]

\( x \) = 0, 2

\( 0^3 - 3(0)^2 - 9 = -9 \)

\( 2^3 - 3(2)^2 - 9 = 8 - 12 - 9 = -13 \)
U) Suppose $u$ and $v$ are differentiable functions at $x = 2$ and $u(2) = 3$, $u'(2) = 3$, $v(2) = 1$, $v'(2) = 2$.

i) Find \( \frac{d}{dx}(uv) = u\left(\frac{dv}{dx}\right) + v\left(\frac{du}{dx}\right) = u \cdot v' + v \cdot u' \)

\[ \frac{d}{dx}(uv) = 9 \]

ii) Find \( \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \cdot u' - u \cdot v'}{v^2} \)

\[ \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{(1)(3) - 3(2)}{1^2} \]

(iii) Find \( \frac{d}{dx}(3u - 2v + 2uv) \)

V) Find the derivative of $y = x$ with respect to $x$

W) Find the derivative of $y = x$ with respect to $t$

X) Find the derivative of $y = x$ with respect to $P$