What you'll Learn About

- How to find the derivative of:
- Functions with positive and negative integer powers
- Functions with products and quotients

A) Using a definition of the derivative find the derivative of $y = x^2$ at $x = a$

$$f'(a) = \lim_{x \to a} \frac{x^2 - a^2}{x - a} = \frac{(x+a)\cancel{(x-a)}}{\cancel{x-a}} = 2a$$

B) Using a definition of the derivative find the derivative of $y = x^3$ at $x = a$

$$f'(a) = \lim_{x \to a} \frac{x^3 - a^3}{x - a} = \frac{\cancel{(x-a)}(x^2 + xa + a^2)}{\cancel{x-a}} = a^2 + a^2 + a^2 = 3a^2$$

C) Using a definition of the derivative find the derivative of $y = x^2 + 4$ at $x = a$

$$f'(a) = \lim_{x \to a} \frac{x^2 + 4 - (a^2 + 4)}{x - a} = \frac{x^2 - a^2}{x - a} = 2a$$

Power Rule

$\boxed{f(x) = x^2 \quad f'(x) = 2x}$

$\boxed{f(x) = x^3 \quad f'(x) = 3x^2}$

$\boxed{f(x) = x^2 + 4 \quad f'(x) = 2x}$

$$\boxed{y' = \frac{4}{5}x^3 + 21x^6}$$

$$y = \frac{1}{5}x^4 + 3x^7$$

$$\boxed{\frac{d}{dx}(x) = 1 \qquad \frac{d}{dx}(\text{constant}) = 0}$$

Find the derivative using the power rule

D) $f(x) = 3 + x^2 - x^3 + x^5$

$$f(x) = 3 + x^2 - x^3 + x^5$$

$$f'(x) = 0 + 2x - 3x^2 + 5x^4$$

E) $y = \frac{x^4}{5} + 3x^7$

$$\boxed{y = \frac{x^4}{5} + 3x^7}$$

F) $y = x^{-3}$

$$y = x^{-3}$$

$$y' = -3x^{-4} = \frac{-3}{x^4}$$

G) $y = \frac{x^{-5}}{3} + \frac{x^{-3}}{4} - \frac{1}{x}$

$$y = \frac{1}{3}x^{-5} + \frac{1}{4}x^{-3} - x^{-1}$$

$$y' = \frac{-5}{3}x^{-6} - \frac{3}{4}x^{-4} + x^{-2}$$

$$\sqrt{x} = x^{1/2}$$

H) $f(x) = 4\sqrt{x} - \frac{1}{x} + \frac{2}{\sqrt{x}}$

$$f(x) = 4\sqrt{x} - \frac{1}{x} + \frac{2}{\sqrt{x}}$$

$$f(x) = 4x^{1/2} - x^{-1} + 2x^{-1/2}$$

$$-\frac{1}{2} - 1$$

$$-\frac{1}{2} - \frac{2}{2}$$

$$f'(x) = 2x^{-1/2} + x^{-2} - 1x^{-3/2} = \frac{2}{\sqrt{x}} + \frac{1}{x^2} - \frac{1}{\sqrt{x^3}}$$

Find the $\boxed{\text{Horizontal Tangents}}$ of each curve

Horizontal Tangent

* slope = 0
* derivative = 0
* always occur at maximums/min (possible)

I) $x^3 + 2x^2 =$

$$y = x^3 + 2x^2$$

$$\frac{dy}{dx} = 3x^2 + 4x$$

$$0 = 3x^2 + 4x$$

$$0 = x(3x + 4)$$

$$0 = x \qquad 3x + 4 = 0 \qquad x = -\frac{4}{3}$$

J) $\frac{2}{3}x^3 - \frac{5}{2}x^2 - 3x = y$

$$y = \frac{2}{3}x^3 - \frac{5}{2}x^2 - 3x$$

$$\frac{dy}{dx} = 2x^2 - 5x - 3$$

$$0 = 2x^2 - 5x - 3$$

$$0 = (2x + 1)(x - 3)$$

$$x = -\frac{1}{2} \qquad x = 3$$