

Find the limit of the functions that involve  $e^x$

$$3. \lim_{x \rightarrow \infty} \frac{e^{-x}}{x} = \frac{1}{xe^x} = 0$$

Horizontal  
Asy

$$\lim_{x \rightarrow -\infty} \frac{e^{-x}}{x} = -\infty$$

$$A) \lim_{x \rightarrow \infty} \frac{e^x + 2x}{2x} = \infty$$

$$\lim_{x \rightarrow \infty} \frac{e^x + 2x}{2x}$$

$$\lim_{x \rightarrow \infty} \frac{e^x}{2x} + \frac{2x}{2x}$$

$$B) \lim_{x \rightarrow -\infty} \frac{e^x + 2x}{2x} =$$

$$\lim_{x \rightarrow -\infty} \frac{e^x + 2x}{2x}$$

$$\lim_{x \rightarrow -\infty} \frac{e^x}{2x} + \frac{2x}{2x} = 0 + 1 = 1$$

Horizontal  
Asy

Find the limit of the functions that involve sine and cosine

$$C) \lim_{x \rightarrow -\infty} \frac{x^3 + \cos x}{x^3}$$

$$\lim_{x \rightarrow -\infty} \frac{x^3}{x^3} + \frac{\cos x}{x^3}$$

$$1 + 0$$

$$1$$

$$E) \lim_{x \rightarrow \infty} \sin\left(\frac{1}{x}\right)$$

$$\lim_{x \rightarrow \infty} \sin(0) = 0$$

$$D) \lim_{x \rightarrow +\infty} \frac{x^3 + \cos x}{x^3}$$

$$\lim_{x \rightarrow +\infty} \frac{x^3}{x^3} + \frac{\cos x}{x^3}$$

$$1 + 0 = 1$$

$$F) \lim_{x \rightarrow \infty} \frac{\sin\left(\frac{1}{x}\right)}{1 + \frac{1}{x}} = \frac{\sin(0)}{1 + 0} = \frac{0}{1} = 0$$



Find the limit of the functions that involve absolute value

$$8A) \lim_{x \rightarrow \infty} \frac{5x-2}{|x|-1} = \frac{5x-2}{x-1} = 5$$

$$8B) \lim_{x \rightarrow -\infty} \frac{5x-2}{|x|-1} = -5$$

53A) Find the limit of  $f(x)$  as

a)  $x \rightarrow -\infty$ , b)  $x \rightarrow \infty$ , c)  $x \rightarrow 0^+$ , d)  $x \rightarrow 0^-$  e)  $x \rightarrow 1^-$  f)  $x \rightarrow 1^+$

a)  $\lim_{x \rightarrow -\infty} \frac{3x-1}{2x+5} = \frac{3}{2}$

b)  $\lim_{x \rightarrow \infty} \frac{2}{x-1} = 0$

c)  $\lim_{x \rightarrow 0^+} \frac{2}{x-1} = \frac{2}{0-1} = -2$

d)  $\lim_{x \rightarrow 0^-} \frac{3x-1}{2x+5} = -\frac{1}{5}$

e)  $\lim_{x \rightarrow 1^-} \frac{2}{x-1} = \frac{2}{0} \rightarrow \infty$

$x=0.9 \quad y = \frac{2}{-0.1}$

f)  $\lim_{x \rightarrow 1^+} \frac{2}{x-1} = \frac{2}{0} \rightarrow \infty$

$x=1.1 \quad y = \frac{2}{0.1}$

\* If  $f(x) = \begin{cases} \frac{3x-1}{2x+5} & x < 0 \\ \frac{2}{x-1} & x \geq 0 \end{cases}$  Left of  $x=0$

Right of  $x=0$

55A) Sketch a graph of a function that satisfies the following conditions

$(0, 2) \leftarrow \lim_{x \rightarrow 0} f(x) = 2$

$\lim_{x \rightarrow -2^-} f(x) = \infty$

$\lim_{x \rightarrow -2^+} f(x) = -\infty$

$\lim_{x \rightarrow -\infty} f(x) = 2$

$\lim_{x \rightarrow \infty} f(x) = -3$

$\lim_{x \rightarrow 3^-} f(x) = \infty$

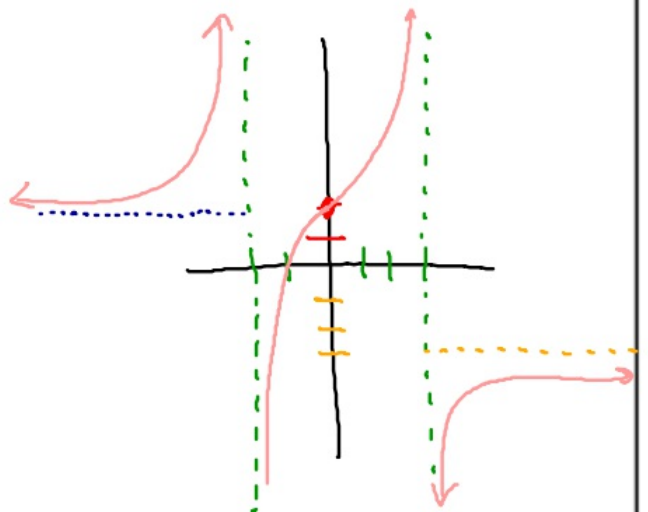
$\lim_{x \rightarrow 3^+} f(x) = -\infty$

V.A.  $x = -2 \leftarrow$

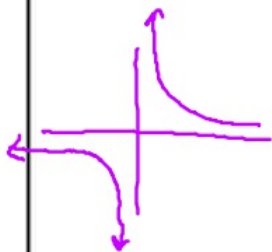
H.A.  $y = 2 \leftarrow$

H.A.  $y = -3 \leftarrow$

V.A.  $x = 3 \leftarrow$



$$y = \frac{1}{x}$$



Find the limit of the functions using the sandwich theorem

Example 9 (p. 65)

$$\lim_{x \rightarrow 0} \left[ x^2 \sin\left(\frac{1}{x}\right) \right]$$

Between -1 and 1

0

$$\lim_{x \rightarrow 0} x^2 \sin\left(\frac{1}{x}\right) = 0$$

p. 76

$$10) \lim_{x \rightarrow -\infty} \frac{1 - \cos x}{x^2} = 0$$

$$12) \lim_{x \rightarrow \infty} \frac{\sin x^2}{x} = 0$$