What you'll Learn About

- Continuity at a point
- Continuous Functions
- Intermediate Value Theorem for Continuous Functions

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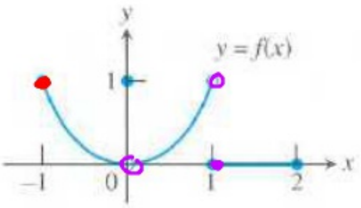
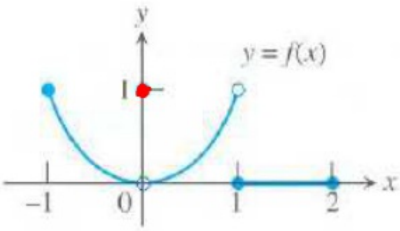


1a. Does $f(-1)$ exist? yes

1b. Does $\lim_{x\to -1^+} f(x)$ exist? yes

1c. Does $\lim_{x\to -1^+} f(x) = f(-1)$ yes

1d. Is f continuous at x = -1? yes



2a. Does $f(0)$ exist? yes

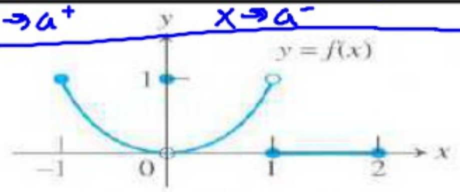
$2b_i$. Does $\lim_{x\to 0^+} f(x)$ exist? yes

$2b_{ii}$. Does $\lim_{x\to 0^-} f(x)$ exist? yes

2c. Does $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^-} f(x) \neq f(0)$ NO

2d. Is f continuous at x = 0? NO (Removeable)

A function is continuous at $x=a$ if $\lim_{x\to a^+} f(x) = \lim_{x\to a^-} f(x) = f(a)$ *



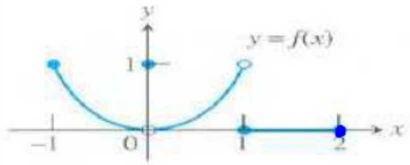
$3a.$ Does $f(1)$ exist? yes $f(1)=0$

$3b_i.$ Does $\lim_{x\to 1^+} f(x)$ exist? 0 yes

$3b_{ii}.$ Does $\lim_{x\to 1^-} f(x)$ exist? 1 yes

$3c.$ Does $\lim_{x\to 1^+} f(x) = \lim_{x\to 1^-} f(x) = f(1)$ NO

$3d.$ Is f continuous at $x=1$? NO Jump

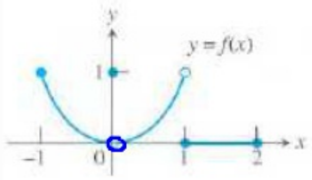


$4a.$ Does $f(2)$ exist? yes

$4b.$ Does $\lim_{x\to 2^-} f(x)$ exis yes

$4c.$ Does $\lim_{x\to 2^-} f(x) = f(2)$ yes
$0 = 0$

$4d.$ Is f continuous at $x=2$? yes



5. For what values is the function continuous $x \neq 0, 1$

$6a.$ Is it possible to extend f to be continuous at $x=0$? If so, what value should the extendec function have? If not, why not? $f(0)=0$

$6b.$ Is it possible to extend f to be continuous at $x=1$? If so, what value should the extended function have? If not, why not? Not a jump

Removeable
Jump
Vertical asy

Determine the type of discontinuity

~~Removeable~~, Jump (If limits are diff)

Left / Right

$$A)\ f(x) = \begin{cases} 3+x & x<2 \\ 1 & x=2 \\ \dfrac{x}{2} & x>2 \end{cases}$$

$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x) = f(2)$

$5 \neq 1 = 1$

$$B)\ f(x) = \begin{cases} \dfrac{1}{x-2} & x<2 \\ x^2+5x & x>2 \end{cases}$$

$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x) = f(2)$

V.A

$\frac{1}{0} \neq 14 \neq$ DNE

$$C)\ f(x) = \begin{cases} 9-x^2 & x\neq 3 \\ 5 & x=3 \end{cases}$$

$\lim_{x\to 3^-} f(x) = \lim_{x\to 3^+} f(x) = f(3)$

Removeable

$0 = 0 \neq 5$

$$D)\ f(x) = \begin{cases} 6-x & x<3 \\ 2x-3 & x>3 \end{cases}$$

$\lim_{x\to 3^-} f(x) = \lim_{x\to 3^+} f(x) = f(3)$

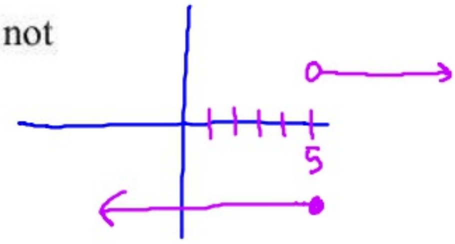
$3 = 3 \neq$ DNE

Hole
Removeable

Given the following information, sketch a graph of f(x)

A) f(x) exists, but $\lim\limits_{x \to 5}$ does not

Jump

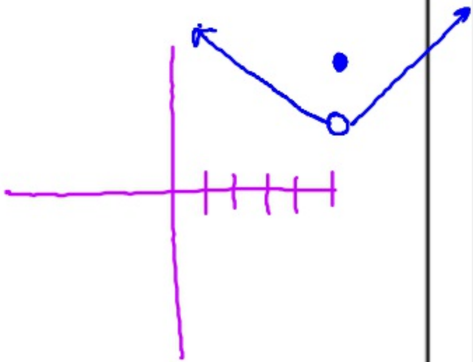


B) f(5) exists

$\lim\limits_{x \to 5}$ exists

Hole

f is not continuous at $x = 5$



Find a value for a so that the function is continuous

47) $f(x) = \begin{cases} x^2 - 1 & x < 3 \\ 2ax & x \geq 3 \end{cases}$

At x = 3 both functions have to have the same y-value

(Both functions = to each other)

$x = 3$

$x^2 - 1 = 2ax$

$9 - 1 = 6a$

$\frac{8}{6} = \frac{6a}{6}$

$\frac{4}{3} = a$