What you'll Learn About
- Continuity at a point
- Continuous Functions
- Intermediate Value Theorem for Continuous Functions

1a. Does \( f(-1) \) exist? \( \text{yes} \)
1b. Does \( \lim_{x \to -1} f(x) \) exist? \( \text{yes} \)
1c. Does \( \lim_{x \to -1} f(x) = f(-1) \)? \( \text{yes} \)
1d. Is \( f \) continuous at \( x = -1 \)? \( \text{yes} \)

2a. Does \( f(0) \) exist? \( \text{yes} \)
2b. Does \( \lim_{x \to 0} f(x) \) exist? \( \text{yes} \)
2b. Does \( \lim_{x \to 0^+} f(x) \) exist? \( \text{yes} \)
2c. Does \( \lim_{x \to 0^+} f(x) = \lim_{x \to 0^-} f(x) \neq f(0) \)? \( \text{NO} \)
2d. Is \( f \) continuous at \( x = 0 \)? \( \text{NO (Removable)} \)
A function is continuous at \( x = a \) if \( \lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) = f(a) \)

3a. Does \( f(1) \) exist? \( \text{Yes} \) \( f(1) = 0 \)

3b. Does \( \lim_{x \to 1^-} f(x) \) exist? \( \text{Yes} \)

3c. Does \( \lim_{x \to 1^+} f(x) = \lim_{x \to 1} f(x) = f(1) \)? \( \text{No} \) \( \text{Jump} \)

3d. Is \( f \) continuous at \( x = 1 \)? \( \text{No} \) \( \text{Jump} \)

4a. Does \( f(2) \) exist? \( \text{Yes} \)

4b. Does \( \lim_{x \to 2} f(x) \) exist? \( \text{Yes} \)

4c. Does \( \lim_{x \to 2} f(x) = f(2) \)? \( 0 = 0 \) \( \text{Yes} \)

4d. Is \( f \) continuous at \( x = 2 \)? \( \text{Yes} \)

5. For what values is the function continuous \( x \neq 0, 1 \)

6a. Is it possible to extend \( f \) to be continuous at \( x = 0 \)? If so, what value should the extended function have? If not, why not? \( f(b) = 0 \)

6b. Is it possible to extend \( f \) to be continuous at \( x = 1 \)? If so, what value should the extended function have? If not, why not? \( \text{Not a Jump} \)
Determine the type of discontinuity

A) \( f(x) = \begin{cases} 
3 + x & x < 2 \\
1 & x = 2 \\
\frac{x}{2} & x > 2 
\end{cases} \)

Left \( \lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x) = f(2) \)

Right \( \lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x) = f(2) \)

Removable

\( x = 2 \)

\( 5 \neq 1 = 1 \)

B) \( f(x) = \begin{cases} 
\frac{1}{x - 2} & x < 2 \\
x^2 + 5x & x > 2 
\end{cases} \)

\( \lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x) = f(2) \)

\( \frac{x^2}{x - 2} \neq 1 \neq \text{DNE} \)

C) \( f(x) = \begin{cases} 
9 - x^2 & x \neq 3 \\
5 & x = 3 
\end{cases} \)

\( \lim_{x \to 3^-} f(x) = \lim_{x \to 3^+} f(x) = f(3) \)

Removable

\( 0 = 0 \neq 5 \)

D) \( f(x) = \begin{cases} 
6 - x & x < 3 \\
2x - 3 & x > 3 
\end{cases} \)

\( \lim_{x \to 3^-} f(x) = \lim_{x \to 3^+} f(x) = f(3) \)

\( 3 = 3 \neq \text{DNE} \)

Hole

Removable
Given the following information, sketch a graph of \( f(x) \)

**A)** \( f(x) \) exists, but \( \lim_{x \to 5} f(x) \) does not

\[ \text{Jump} \]

**B)** \( f(5) \) exists

\[ \lim_{x \to 5} f(x) \text{ exists} \]

\( f \) is not continuous at \( x = 5 \)

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Find a value for \( a \) so that the function is continuous

47) \( f(x) = \begin{cases} 
  x^2 - 1 & \text{if } x < 3 \\
  2ax & \text{if } x \geq 3 
\end{cases} \)

At \( x = 3 \) both functions have to have the same \( y \)-value.

(Both functions = to each other)

\[ \frac{x^2 - 1}{2ax} = \]

\[ \frac{9 - 1}{6a} = \]

\[ \frac{8}{6} = \]

\[ \frac{4}{3} = a \]