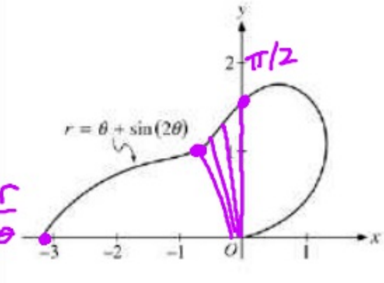
Page 14



2005 BC2 (Calculator)

The curve above is drawn in xy-plane and is described by the equation in polar coordinates $r = \theta + \sin(2\theta)$ for $0 \le \theta \le \pi$, where r is measured in meters and θ is measured in radians. The derivative of r with respect to θ is given by $\frac{dr}{d\theta} = 1 + 2\cos(2\theta)$.

$$\frac{dx}{d\theta} = r(-\sin\theta) + \cos\theta \frac{dr}{d\theta}$$

a. Find the slope of the curve at the point $\theta = \frac{\pi}{2}$.

$x = r\cos\theta$

$x = (\theta + \sin 2\theta)\cos\theta$

$y = r\sin\theta$

$y = (\theta + \sin 2\theta)\sin\theta$

$$\frac{dy}{dx} = \frac{-1}{(-\pi/2)}$$

c. Find the angle θ that corresponds to the point on the curve with x-coordinate -2.

$x = r\cos\theta$

$-2 = (\theta + \sin 2\theta)\cos\theta$

$\theta = 2.786$

d. For $\frac{\pi}{2} < \theta \le \frac{2\pi}{3}$, $\frac{dr}{d\theta}$ is negative. What does this fact say about r? What does this fact say about the curve?

radios is getting smaller

$\frac{dr}{d\theta}$ and r are opp signs ← closer to the origin

e. Find the value of θ in the interval $0 \le \theta \le \frac{\pi}{2}$ that corresponds to the point on the curve in the first quadrant with greatest distance from the origin. Justify your answer.

$\boxed{r = \theta + \sin 2\theta}$

$\frac{dr}{d\theta} = 1 + 2\cos 2\theta$

$0 = 1 + 2\cos 2\theta$

$-\frac{1}{2} = \cos 2\theta$

$2\theta = \frac{2\pi}{3}$

$\theta = \frac{\pi}{3}$

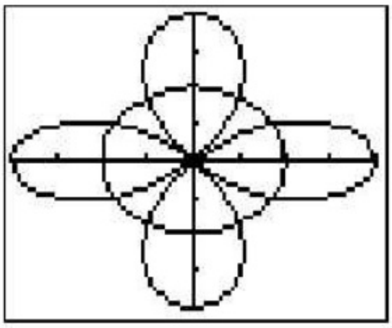
$\boxed{\text{abs max}}$

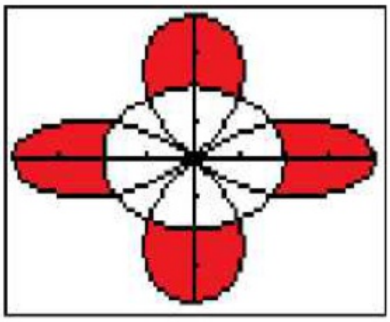
$r(0) = 0$

$r\left(\frac{\pi}{3}\right) = \pi/3 + \sqrt{3}/2 \rightarrow \boxed{\text{abs max}}$

$r\left(\frac{\pi}{2}\right) = \frac{\pi}{2}$

56. Inside the four-petaled rose r = $4\cos 2\theta$ and outside the circle r = 2





Determine the polar curves and shaded area represented by the integral given below.

$$A=\frac{1}{2}\int_{\pi/6}^{5\pi/6}\left(2\sin\theta\right)^{2}d\theta-\frac{1}{2}\int_{\pi/6}^{5\pi/6}\left(1\right)^{2}d\theta$$