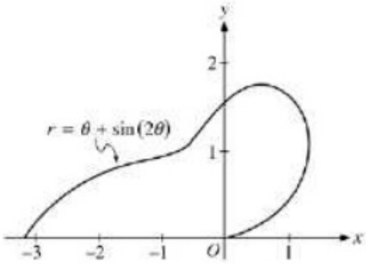


2005 BC2 (Calculator)

The curve above is drawn in xy-plane and is described by the equation in polar coordinates $r = \theta + \sin(2\theta)$ for $0 \le \theta \le \pi$, where r is measured in meters and θ is measured in radians. The derivative of r with respect to θ is given by $\frac{dr}{d\theta} = 1 + 2\cos(2\theta)$.

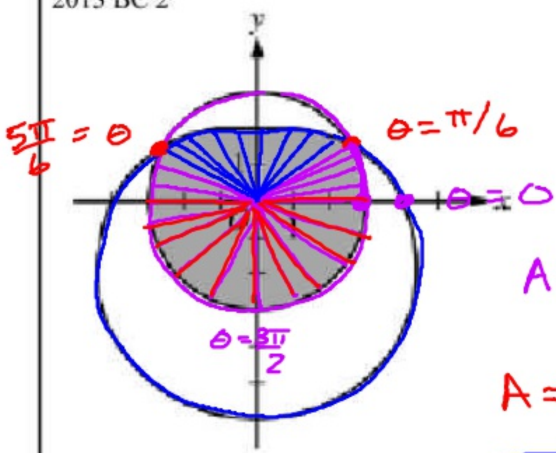
a. Find the slope of the curve at the point $\theta = \frac{\pi}{2}$.

c. Find the angle θ that corresponds to the point on the curve with x-coordinate -2.

d. For $\frac{\pi}{2} < \theta \le \frac{2\pi}{3}$, $\frac{dr}{d\theta}$ is negative. What does this fact say about r? What does this fact say about the curve?

e. Find the value of θ in the interval $0 \le \theta \le \frac{\pi}{2}$ that corresponds to the point on the curve in the first quadrant with greatest distance from the origin. Justify your answer.

2013 BC 2



$$A = \frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}}(4-2\sin\theta)^2$$

$$A = \frac{1}{2}(2)\int_0^{\pi/6}(3)^2 =$$

$$A = \frac{1}{2}\pi(3)^2 =$$

The graphs of the polar curves r = 3 and $r = 4 - 2\sin\theta$ are shown in the figure above. The curves intersect when $\theta = \frac{\pi}{6}$ and $\theta = \frac{5\pi}{6}$.

a. Let S be the shaded region that is inside the graph of r = 3 and also inside the graph of $r = 4 - 2\sin\theta$. Find the area of S.

b. A particle moves along the polar curve $r = 4 - 2\sin\theta$ so that at time t seconds, $\theta = t^2$. Find the time t in the interval $1 \le t \le 2$ for which the x-coordinate of the particle's position is -1.

c. For the particle described in part (b), find the position vector in terms of t. Find the velocity vector at time t = 1.5.

54. Shared by the cardiods $r = 2(1 + \cos\theta)$ and $r = 2(1 - \cos\theta)$

$$2(1+\cos\theta)=2(1-\cos\theta)$$

$$1+\cos\theta = 1-\cos\theta$$

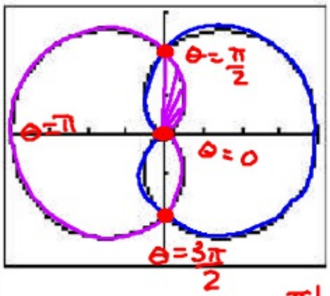
$$\cos\theta = -\cos\theta$$

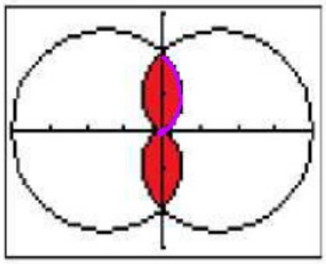
$$+\cos\theta \quad +\cos\theta$$

$$2\cos\theta = 0$$

$$\cos\theta = 0$$

$$\theta = \frac{\pi}{2}, \frac{3\pi}{2}$$





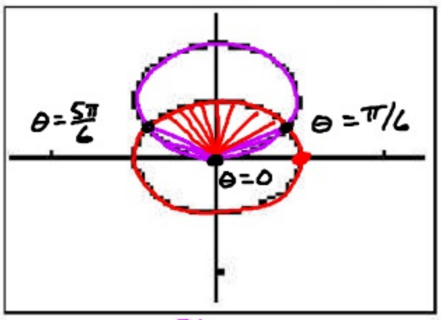
$$A = \frac{1}{2}(4)\int_0^{\pi/2} \left(2(1-\cos\theta)\right)^2$$

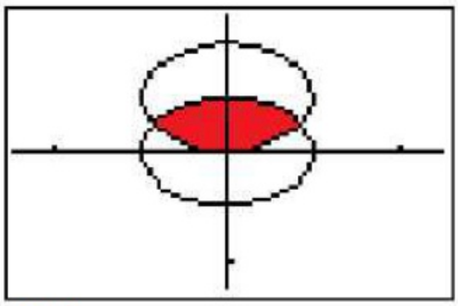
52. Shared by $r = 1$ and $r = 2\sin\theta$

$$1 = 2\sin\theta$$

$$\frac{1}{2} = \sin\theta$$

$$\theta = \frac{\pi}{6}, \frac{5\pi}{6}$$





$$A = \frac{1}{2}(2)\int_0^{\pi/6} (2\sin\theta)^2\, d\theta + \frac{1}{2}\int_{\pi/6}^{5\pi/6} 1\, d\theta =$$

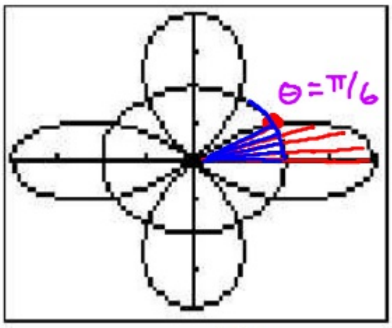
$4\cos 2\theta = 2$

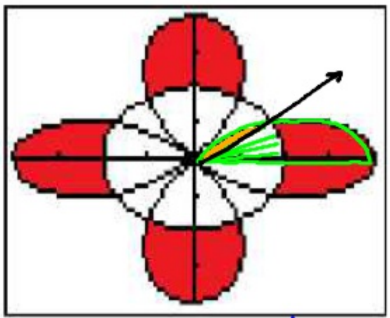
$\cos 2\theta = \frac{1}{2}$

$2\theta = \frac{\pi}{3}$

$\theta = \frac{\pi}{6}$

56. Inside the four-petaled rose r = 4cos 2θ and outside the circle r = 2





$$A = \frac{1}{2}(8)\int_0^{\pi/6} (4\cos 2\theta)^2 \, d\theta - \frac{1}{2}(8)\int_0^{\pi/6} (2)^2 \, d\theta$$

Determine the polar curves and shaded area represented by the integral given below.

$$A = \frac{1}{2}\int_{\pi/6}^{5\pi/6} (2\sin\theta)^2 \, d\theta - \frac{1}{2}\int_{\pi/6}^{5\pi/6} (1)^2 \, d\theta$$