2005 BC2 (Calculator)

The curve above is drawn in xy-plane and is described by the equation in polar coordinates $r = \theta + \sin(2\theta)$ for $0 \leq \theta \leq \pi$, where $r$ is measured in meters and $\theta$ is measured in radians. The derivative of $r$ with respect to $\theta$ is given by $\frac{dr}{d\theta} = 1 + 2\cos(2\theta)$.

a. Find the slope of the curve at the point $\theta = \frac{\pi}{2}$.

c. Find the angle $\theta$ that corresponds to the point on the curve with x-coordinate -2.

d. For $\frac{\pi}{2} < \theta \leq \frac{2\pi}{3}$, $\frac{dr}{d\theta}$ is negative. What does this fact say about $r$? What does this fact say about the curve?

e. Find the value of $\theta$ in the interval $0 \leq \theta \leq \frac{\pi}{2}$ that corresponds to the point on the curve in the first quadrant with greatest distance from the origin. Justify your answer.
The graphs of the polar curves $r = 3$ and $r = 4 - 2\sin\theta$ are shown in the figure above. The curves intersect when $\theta = \frac{\pi}{6}$ and $\theta = \frac{5\pi}{6}$.

a. Let S be the shaded region that is inside the graph of $r = 3$ and also inside the graph of $r = 4 - 2\sin\theta$. Find the area of S.

b. A particle moves along the polar curve $r = 4 - 2\sin\theta$ so that at time t seconds, $\theta = t^2$. Find the time t in the interval $1 \leq t \leq 2$ for which the x-coordinate of the particle's position is -1.

c. For the particle described in part (b), find the position vector in terms of t. Find the velocity vector at time $t = 1.5$. 

$A = \frac{1}{2} \int_{\pi/6}^{5\pi/6} (4 - 2\sin\theta)^2 \, d\theta = \frac{1}{2} \int_{0}^{\pi/6} (3)^2 \, d\theta$
54. Shared by the cardioid $r = 2(1 + \cos \theta)$ and $r = 2(1 - \cos \theta)$

\[ A = \frac{1}{2} \left( \int_0^{\pi/2} (2(1 - \cos \theta))^2 \, d\theta + \int_0^{\pi/2} 1 \, d\theta \right) \]

52. Shared by $r = 1$ and $r = 2\sin \theta$

\[ A = \frac{1}{2} \left( \int_0^{\pi/6} (2\sin \theta)^2 \, d\theta + \int_0^{\pi/6} 1 \, d\theta \right) \]
4 \cos 2\theta = 2
\cos 2\theta = \frac{1}{2}
2\theta = \frac{\pi}{3}
\theta = \frac{\pi}{6}

\[ A = \frac{1}{2} \left( \theta \right) \int_{0}^{\pi/6} \left( 4 \cos 2\theta \right)^2 \, d\theta = \frac{1}{2} \left( \theta \right) \int_{0}^{\pi/6} (2)^2 \, d\theta \]

Determine the polar curves and shaded area represented by the integral given below.

\[ A = \frac{1}{2} \int_{\pi/6}^{5\pi/6} \left( 2 \sin \theta \right)^2 \, d\theta - \frac{1}{2} \int_{\pi/6}^{5\pi/6} (1)^2 \, d\theta \]