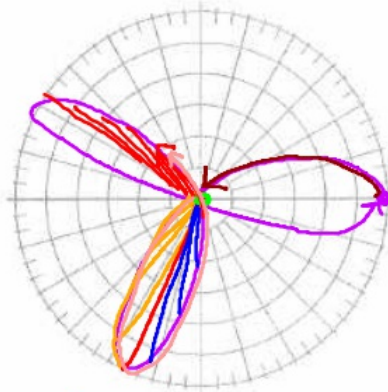


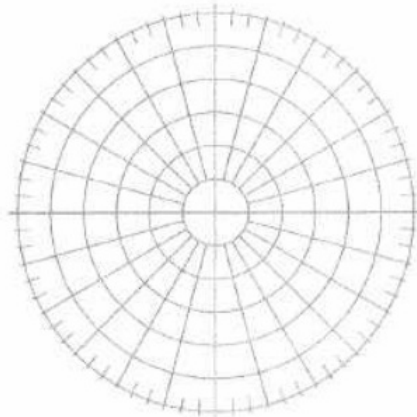
$$r = \cos 3\theta$$

$$\frac{dr}{d\theta} = 3$$
$$\theta = \frac{\pi}{2}$$

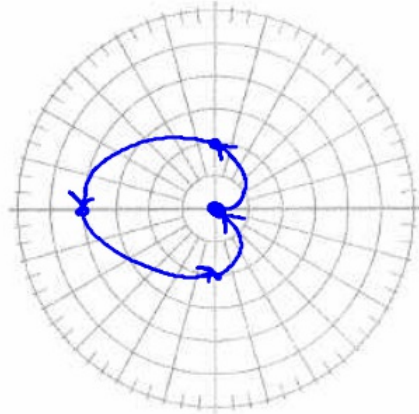
5. Graph the circle  $r = 2\sin\theta$ . Then determine the slope of the curve at  $\theta = \frac{\pi}{4}$  and then determine  $\frac{dr}{d\theta}$



6. Graph the cardioid  $r = 2 - 2\sin\theta$ . Then determine the slope of the curve at  $\theta = \frac{\pi}{2}$  and then determine  $\frac{dr}{d\theta}$



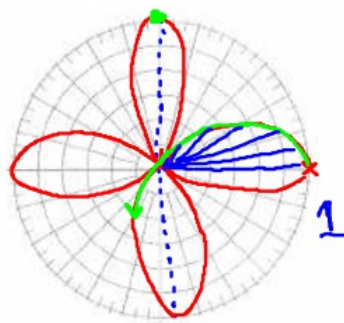
7. Graph the cardioid  $r = 2 - 2\cos\theta$ . Then determine the slope of the curve at  $\theta = \frac{\pi}{2}$



8. Graph the 4-petaled rose  $r = \cos 2\theta$ . Then determine the slope of the curve at

$\theta = \frac{3\pi}{2}$  and then determine  $\frac{dr}{d\theta} = -2\sin 2\theta \rightarrow -2\sin 2\left(\frac{3\pi}{2}\right)$

$$\theta = \frac{3\pi}{2} \rightarrow -2\sin 3\pi = 0$$



$$\frac{dr}{d\theta} = 0$$

$$\frac{dy}{dx} = \frac{0}{-1} = 0$$

$$x = r \cos\theta$$

$$y = r \sin\theta$$

$$x = \cos 2\theta \cos\theta$$

$$y = \cos 2\theta \sin\theta$$

$$\frac{dx}{d\theta} = \cos(2\theta)(-\sin\theta) + \cos\theta(-2\sin 2\theta)$$

$$\frac{dy}{d\theta} = \cos 2\theta \cdot \cos\theta + \sin\theta(-2\sin 2\theta)$$

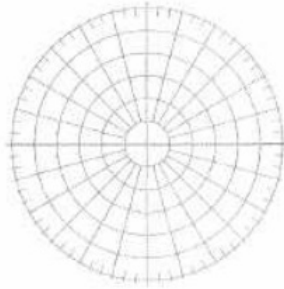
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$$= \cos\left(2 \cdot \frac{3\pi}{2}\right) \cdot \left(-\sin\left(\frac{3\pi}{2}\right)\right) = -1$$

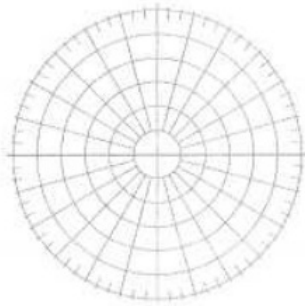
$$\frac{dy}{d\theta} = 0$$

$$\cos \frac{3\pi}{2}$$

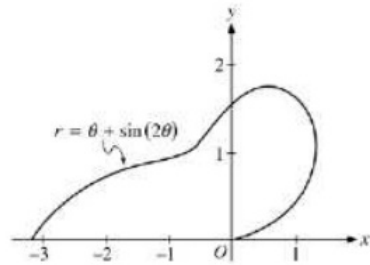
1. Graph the 4 pedaled rose  $r = \sin 2\theta$ . Then determine the slope of the curve at  $\theta = \frac{3\pi}{2}$



2. Graph the limaçon  $r = 2 - 3\sin\theta$ . Then determine the slope of the curve at  $\theta = \pi$



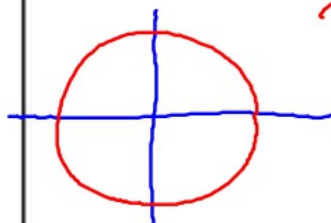
2005 BC2 (Calculator)



The curve above is drawn in  $xy$ -plane and is described by the equation in polar coordinates  $r = \theta + \sin(2\theta)$  for  $0 \leq \theta \leq \pi$ , where  $r$  is measured in meters and  $\theta$  is measured in radians. The derivative of  $r$  with respect to  $\theta$  is given by  $\frac{dr}{d\theta} = 1 + 2\cos(2\theta)$ .

- a. Find the slope of the curve at the point  $\theta = \frac{\pi}{2}$ .
  
  
  
  
  
  
  
  
  
  
- c. Find the angle  $\theta$  that corresponds to the point on the curve with  $x$ -coordinate  $-2$ .
  
  
  
  
  
  
  
  
  
  
- d. For  $\frac{\pi}{2} < \theta \leq \frac{2\pi}{3}$ ,  $\frac{dr}{d\theta}$  is negative. What does this fact say about  $r$ ? What does this fact say about the curve?
  
  
  
  
  
  
  
  
  
  
- e. Find the value of  $\theta$  in the interval  $0 \leq \theta \leq \frac{\pi}{2}$  that corresponds to the point on the curve in the first quadrant with greatest distance from the origin. Justify your answer.

Circle  $r = 4$



Whole Circle

$$A = \pi r^2$$

$$A = \pi(4)^2 = 16\pi$$

$\frac{1}{2}$  circle

$$A = \left(\frac{1}{2}\right)\pi r^2 = 8\pi$$

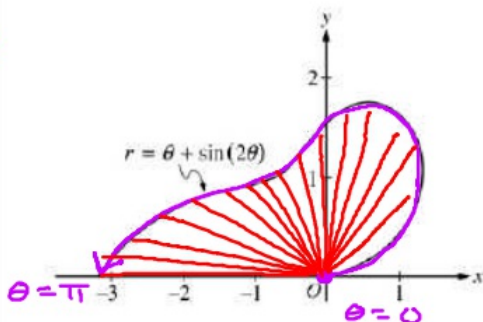
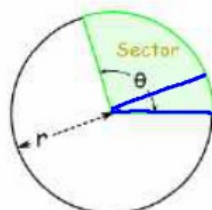
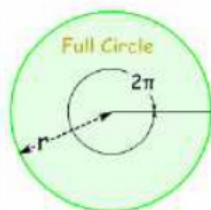
$$\frac{\frac{1}{2}\pi}{2\pi} =$$

## Polar Area

### Area of a Sector

You can work out the Area of a Sector by comparing its angle to the angle of a full circle.

Note: we are using radians for the angles.



The curve above is drawn in  $xy$ -plane and is described by the equation in polar coordinates  $r = \theta + \sin(2\theta)$  for  $0 \leq \theta \leq \pi$ , where  $r$  is measured in meters and  $\theta$  is measured in radians. The derivative of  $r$  with respect to  $\theta$  is given by

$$\frac{dr}{d\theta} = 1 + 2\cos(2\theta).$$

b. Find the area bounded by the curve and the  $x$ -axis

$$A = \frac{1}{2} \int_0^{\pi} (\theta + \sin(2\theta))^2 d\theta =$$

$$A = \frac{1}{2} \int_0^{\pi} 4^2 d\theta = \frac{1}{2} [4\theta]_0^{\pi}$$

$$\frac{1}{2} [16\pi] = 8\pi$$

$$A = \left(\frac{\theta}{2\pi}\right) \pi r^2$$

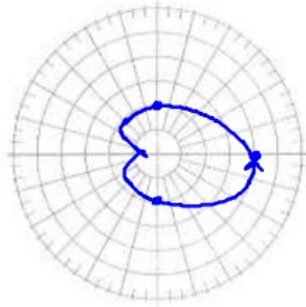
$$A = \frac{\theta}{2} r^2$$

$$A = \frac{1}{2} \theta r^2$$

$$A = \frac{1}{2} \int_{\theta_1}^{\theta_2} r^2 d\theta$$

$$r = 2 + 2\cos\theta$$

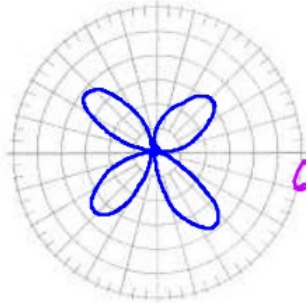
Find the area inside the polar curve  $r = 2(1 + \cos\theta)$ .



$$A = \frac{1}{2} \int_0^{2\pi} [2(1 + \cos\theta)]^2 d\theta$$

$$A = \frac{1}{2} (2) \int_0^{\pi} [2(1 + \cos\theta)]^2 d\theta \quad \text{OR}$$

Find the area inside the polar curve  $r = 2\sin(2\theta)$ .



$$A = \frac{1}{2} \int_0^{2\pi} (2\sin(2\theta))^2 d\theta$$

OR

$$A = \frac{1}{2} (2) \int_0^{\pi} (2\sin(2\theta))^2 d\theta$$

OR

$$A = \frac{1}{2} (4) \int_0^{\pi/2} (2\sin(2\theta))^2 d\theta$$

OR

$$A = \frac{1}{2} (8) \int_0^{\pi/4} (2\sin(2\theta))^2 d\theta$$

$r = 2\sin\theta$

$$A = \frac{1}{2} \int_0^{\pi} (2\sin\theta)^2 d\theta$$