CALCULUS: Graphical, Numerical, Algebraic by Finney, Demana, Watts and Kennedy

10.3: Polar Equations

What you'll Learn About

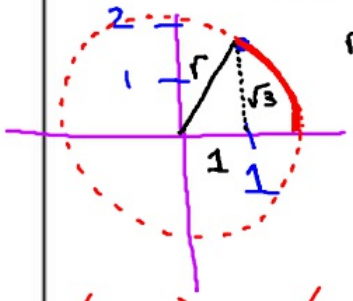
- Calculus involving Polar Equations

1. Plot the rectangular (x, y) coordinate $(1, \sqrt{3})$. Then determine the polar coordinates (r, θ) of the point.

Rectangular → Polar

$r = \sqrt{x^2 + y^2}$

$\theta = \tan^{-1}\left(\frac{y}{x}\right)$



$r^2 = 1^2 + \sqrt{3}^2$

$r = \sqrt{1^2 + \sqrt{3}^2}$

$r = \sqrt{1+3}$

$r = 2$

$\theta = \tan^{-1}\left(\frac{\sqrt{3}}{1}\right)$

$\theta = \tan^{-1}\left(\frac{\sqrt{3}/2}{1/2}\right)$

$\sin\theta = \sqrt{3}/2$ and $\cos\theta = \frac{1}{2}$

$\theta = \frac{\pi}{3}$

$(1, \sqrt{3}) \rightarrow (2, \frac{\pi}{3})$

rect polar

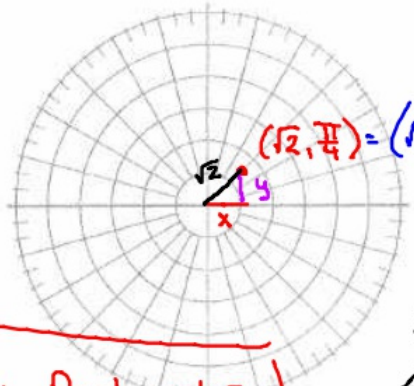
2. Plot the polar (r, θ) coordinate $\left(\sqrt{2}, \frac{\pi}{4}\right)$. Then determine the rectangular coordinate (x, y) of the point.

$\frac{\pi}{4} = 45°$

$\frac{3\pi}{4} = 135°$

$\frac{5\pi}{4} = 225°$

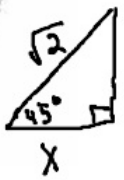
$\frac{7\pi}{4} = 315°$



$(\sqrt{2}, \frac{\pi}{4}) = (\sqrt{2}, -\frac{7\pi}{4}) = (-\sqrt{2}, \frac{5\pi}{4}) = (-\sqrt{2}, -\frac{3\pi}{4})$

Polar to Rectangular

$x = r\cos\theta \quad y = r\sin\theta$



$\cos 45 = \frac{x}{\sqrt{2}}$

$\sqrt{2}\cos 45° = x$

$\sin 45 = \frac{y}{\sqrt{2}}$

$\sqrt{2}\sin 45° = y$

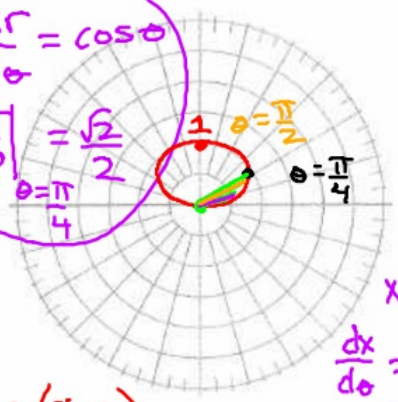
Starts at origin / counterclockwise

3. Graph the circle $r = \sin\theta$. Then determine the slope of the curve at $\theta = \frac{\pi}{4}$

θ	$r = \sin\theta$
0	0
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2} = .707$
$\frac{\pi}{2}$	1

$\frac{dr}{d\theta} = \cos\theta$

$\left.\frac{dr}{d\theta}\right|_{\theta=\frac{\pi}{4}} = \frac{\sqrt{2}}{2}$



Find $\left.\frac{dy}{dx}\right|_{\theta=\frac{\pi}{4}} = \frac{1}{0} \rightarrow$ UND

$x = r\cos\theta \qquad y = r\sin\theta$

$y = \sin\theta(\sin\theta)$

$y = \sin^2\theta$

$x = \sin\theta\cos\theta$

$\frac{dx}{d\theta} = \sin\theta(-\sin\theta) + \cos\theta(\cos\theta)$

$= -\sin^2\theta + \cos^2\theta$

$\frac{dy}{d\theta} = 2\sin\theta\cos\theta$

$\left.\frac{dx}{d\theta}\right|_{\theta=\frac{\pi}{4}} = -\left(\sin\frac{\pi}{4}\right)^2 + \left(\cos\frac{\pi}{4}\right)^2$

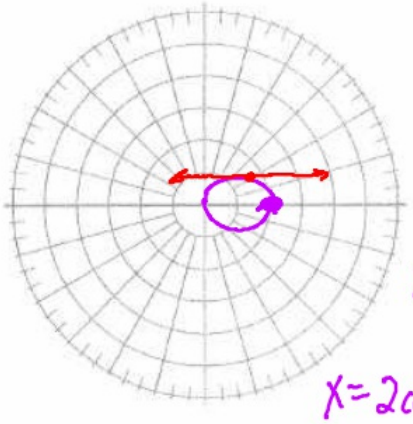
$= -\left(\frac{\sqrt{2}}{2}\right)^2 + \left(\frac{\sqrt{2}}{2}\right)^2$

$= 0$

$\left.\frac{dy}{d\theta}\right|_{\theta=\frac{\pi}{4}} = 2\sin\frac{\pi}{4}\cos\frac{\pi}{4} = 2\cdot\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{2}}{2} = 1$

4. Graph the circle $r = 2\cos\theta$. Then determine the slope of the curve at $\theta = \frac{\pi}{4}$

θ	$r = 2\cos\theta$
0	2
$\frac{\pi}{2}$	0



$\left.\frac{dy}{dx}\right|_{\theta=\frac{\pi}{4}} = \frac{0}{-2} = 0$

$x = r\cos\theta \qquad y = r\sin\theta$

$x = 2\cos\theta\cos\theta \qquad y = 2\cos\theta\sin\theta$

$x = 2\cos^2\theta$