Parametrics and Vectors

18. For what values of t does the curve given by the parametric equations $x = t^3 - t^2 - 1$ and $y = t^4 + 2t^2 - 8t$ have a vertical tangent?

(A) 0 only (B) 0, 2/3, and 1 (C) 1 only (D) No Value (E) 0 and 2/3 only

2. In the xy-plane, the graph of the parametric equations $x = 5t + 2$ and $y = 3t$, for $-3 \le t \le 3$, is a line segment with slope

(A) 3/5 (B) 5/3 (C) 3 (D) 5 (E) 13

4. For $0 \le t \le 13$, an object travels along an elliptical path given by the parametric equations $x = 3\cos t$ and $y = 4\sin t$. At the point where $t = 13$, the object leaves the path and travels along the line tangent to the path at that point. What is the slope of the line on which the object travels?

A) $\frac{-4}{3}$ B) $\frac{-3}{4}$ C) $-\frac{4\tan 13}{3}$ D) $-\frac{4}{3\tan 13}$ E) $-\frac{3}{4\tan 13}$

$\left(\frac{dx}{dt}\right)^2 + \left(3\frac{dx}{dt}\right)^2 = 40 \rightarrow 10\left(\frac{dx}{dt}\right)^2 = 40 \rightarrow \left(\frac{dx}{dt}\right)^2 = 4 \rightarrow \frac{dx}{dt} = 2$

28. In the xy-plane, a particle moves along the parabola $y = x^2 - x$ with a constant speed of $2\sqrt{10}$ units per second. If $\frac{dx}{dt} > 0$, what is the value of $\frac{dy}{dt}$ when the particle is at the point (2, 2)?

A) 2/3 B) $\frac{2\sqrt{10}}{3}$ C) 3 D) 6 E) $6\sqrt{10}$

$\frac{\frac{dy}{dt}}{\frac{dx}{dt}} = 3 \rightarrow \frac{dy}{dt} = 3\frac{dx}{dt}$

$y = x^2 - x \rightarrow \frac{dy}{dx} = 2x - 1 \rightarrow \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = 2x - 1$

Speed $= 2\sqrt{10}$

$\left(\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\right)^2 = \left(2\sqrt{10}\right)^2$

$\frac{dx}{dt} = \sqrt{40 - \left(\frac{dy}{dt}\right)^2}$

(2, 2)

x y

$\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 = 40 \rightarrow \left(\frac{dx}{dt}\right)^2 = 40 - \left(\frac{dy}{dt}\right)^2$