What you'll Learn About
How to take the derivative of a function that is not solved for y (an implicitly defined function)

Implicit Differentiation

Find the derivative of the following function

A) $x^2 + y^2 = 1$

$-x^2 \quad -x^2$

$y^2 = 1 - x^2$

$y = \pm\sqrt{1-x^2} = \pm(1-x^2)^{1/2}$

$\boxed{\frac{dy}{dx}} = \pm\frac{1}{2}(1-x^2)^{-1/2} \cdot -2x = \boxed{\frac{\pm x}{\sqrt{1-x^2}}}$

B) $x = \cos\theta \quad y = \sin\theta$

$\frac{dx}{d\theta} = -\sin\theta \quad \frac{dy}{d\theta} = \cos\theta$

$\frac{dy}{dx} = \frac{\cos\theta}{-\sin\theta}$

C) $x^2 + y^2 = 1$

$(x)^2 + (y)^2$

$2(x)\cdot 1 + 2(y)\frac{dy}{dx} = 0$

$-2x \quad -2x$

$\frac{2y\frac{dy}{dx}}{2y} = \frac{-2x}{2y}$

$\boxed{\frac{dy}{dx} = \frac{-x}{y}}$

D) $x^2 + y^2 = xy$

$2x + 2y\frac{dy}{dx} = x\frac{dy}{dx} + y$

$-2x \quad -x\frac{dy}{dx} \quad -x\frac{dy}{dx} \quad -2x$

$2y\frac{dy}{dx} - x\frac{dy}{dx} = y - 2x$

$\frac{\frac{dy}{dx}(2y - x)}{2y-x} = \frac{y-2x}{2y-x}$

$\boxed{\frac{dy}{dx} = \frac{y-2x}{2y-x}}$

	$E)\ x^2 = \frac{x-y}{x+y}$
	$\cancel{(x+y)^2}\, 2x = \frac{(x+y)\left(1-\frac{dy}{dx}\right) - (x-y)\left(1+\frac{dy}{dx}\right)}{\cancel{(x+y)^2}}\ \cancel{(x+y)^2}$
	$2x(x+y)^2 = \cancel{x} - x\frac{dy}{dx} + y - \cancel{y\frac{dy}{dx}}\ \cancel{-x} - x\frac{dy}{dx} + y + \cancel{y\frac{dy}{dx}}$
	$2x(x+y)^2 = 2y - 2x\frac{dy}{dx}$
	$\frac{2x(x+y)^2}{-2x} - \frac{2y}{-2x} = \frac{-2x\frac{dy}{dx}}{-2x}$ $\quad \boxed{\frac{dy}{dx} = -(x+y)^2 + \frac{y}{x}}$
$x + \tan(xy) = y$	$F)\ x + \tan(xy) = y$
	$1 + \sec^2(xy)\cdot\left[x\frac{dy}{dx} + y\right] = \frac{dy}{dx}$
	$1 + x\sec^2(xy)\frac{dy}{dx} + y\sec^2(xy) = \frac{dy}{dx}$
	$1 + y\sec^2(xy) = \frac{dy}{dx} - x\sec^2(xy)\frac{dy}{dx}$
	$\frac{1 + y\sec^2(xy)}{1 - x\sec^2(xy)} = \frac{\frac{dy}{dx}\left(1 - x\sec^2(xy)\right)}{\left(1 - x\sec^2(xy)\right)}$
	$\boxed{\frac{1 + y\sec^2(xy)}{1 - x\sec^2(xy)} = \frac{dy}{dx}}$