Consider the curve defined by the equation $2y^3 + \cancel{6x^2}y - \cancel{12x}^2 + 6y = 1$ with $\frac{dy}{dx} = \frac{4x - 2xy}{x^2 + y^2 + 1}$

$2y^3 + 6y = 1$

b) Write an equation of each horizontal tangent to the curve

$0 = 4x - 2xy$

$0 = 2x(2 - y)$

$0 = 2x \qquad 2 - y = 0$

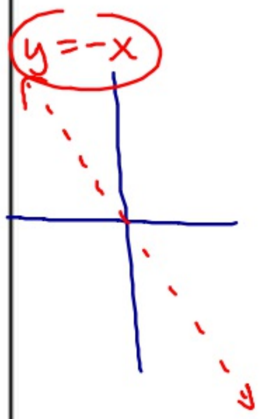
$0 = x \qquad y = 2$

$0 = 4x - 2xy$

$+2xy \qquad +2xy$

$\frac{2xy}{2x} = \frac{4x}{2x}$

$y = 2$



c) The line through the origin with slope -1 is tangent to the curve at point P. Find the x and y-coordinates of P.

$\frac{dy}{dx} = -1 \qquad -1 = \frac{4x - 2xy}{x^2 + y^2 + 1}$

d) Find $\frac{d^2y}{dx^2}$ in terms of x and y.

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What you'll Learn About
How to use derivatives to solve a problem involving rates

A) Water is draining from a cylindrical tank with radius of 15 cm at 3000 cm^3/second. How fast is the surface dropping?

$r = 15$ cm $\frac{dV}{dt} = -3000$ cm^3/sec Find $\frac{dh}{dt}$

$V = \pi r^2 h$ $V = 225\pi h$

$V = \pi (15)^2 h$ $\frac{dV}{dt} = 225\pi \frac{dh}{dt}$

$\underset{cm^3}{(-3000)} = \underset{cm^2}{(225\pi)} \frac{dh}{dt}$

$\boxed{\frac{dh}{dt} = \frac{-3000}{225\pi} \text{ cm/sec}}$

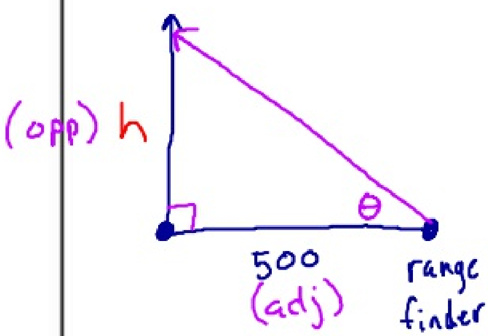
B) A hot-air balloon rising straight up from a level field is tracked by a range finder 500 ft from the lift-off point. At the moment the range finder's elevation angle is 45°, the angle is increasing at the rate of .14 rad/min. How fast is the balloon rising at that moment?

$\theta = 45° = \frac{\pi}{4}$

$\frac{d\theta}{dt} = .14$ rad/min

Find $\frac{dh}{dt}$

Don't use this until we take the derivative



$\frac{h}{500} = \frac{1}{500} h$

$\tan\theta = \frac{h}{500}$

$(500)\ \sec^2\theta \cdot \frac{d\theta}{dt} = \frac{1}{500} \frac{dh}{dt} (500)$

$\boxed{500 \sec^2\left(\frac{\pi}{4}\right)(.14) = \frac{dh}{dt}}$ ✓

$500 \left(\frac{2}{\sqrt{2}}\right)^2 (.14) = \frac{dh}{dt}$

$\boxed{140 \text{ ft/min} = \frac{dh}{dt}}$

① Draw a picture

- only put constants on your picture

② Put everything in symbols

③ Find the equation from the picture to take the derivative of

C) Truck A travels east at 40 mi/hr. Truck B travels north at 30 mi/hr. How fast is the distance between the trucks changing 6 minutes later?

$$V = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dt} = \frac{4}{3}\pi \cdot 3r^2\left(\frac{dr}{dt}\right)$$

D) Water runs into a conical tank at the rate of 9 ft^3/min. The tank stands point down and has a height of 10 ft and a base radius of 5 ft. How fast is the water level rising when the water is 6 ft deep?