

4.5: Linearization

What you'll Learn About
 Linearization is another term for tangent line
 Differentials are part of the derivative
 Mean Value Theorem

Tangent Line

- a) Find the linearization of the function. b) Find $L(a + .1)$ and $f(a + .1)$
 c) Using concavity, determine if the Tangent Line at a is an overestimate or an underestimate. Justify your answer.

1. $f(x) = x^3 - 2x + 3$ $a = 1$

a) $(1, 2)$ $f'(x) = 3x^2 - 2$ $f'(1) = 1$

$L(x) = 2 + 1(x-1)$ ✓

b) $f(1.1) = 2.131$ $L(1.1) = 2.1$

c) $f''(x) = 6x$ $f''(1) = 6 > 0$ $f(x)$ Concave up
 $L(x)$ underestimate

2. $f(x) = x + \frac{1}{x}$ ~~$a = 0$~~ $a = 2$

a) $(2, 2.5)$ $f(x) = x + x^{-1}$
 $f'(2) = \frac{3}{4}$ $f'(x) = 1 - x^{-2} = 1 - \frac{1}{x^2}$

$L(x) = 2.5 + \frac{3}{4}(x-2)$

b) $L(2.1) = 2.5 + .75(.1) = 2.575$

c) $f''(x) = 2x^{-3} = \frac{2}{x^3}$

$f(2.1) = 2.1 + \frac{1}{2.1} = 2.576$

$f''(2) = \frac{2}{8} > 0$ Concave up

$L(x)$ is an underestimate