

- b. For what ticket price does the committee expect an income of zero?
- c. What ticket price will generate the greatest income? How much income is expected at that ticket price?
- d. Use your answers to Parts b and c to sketch a graph of $I = -75p^2 + 950p$.

Adding a Linear Term

5. Study the tables and graphs produced by such functions for several combinations of positive and negative numbers.

Set 1

Set 2

Set 3

$$y = x^2$$

$$y = -x^2$$

$$y = 2x^2$$

$$y = x^2 + 4x$$

$$y = -x^2 + 5x$$

$$y = 2x^2 + 6x$$

$$y = x^2 - 4x$$

$$y = -x^2 - 5x$$

$$y = 2x^2 - 6x$$

$$y = x(x+4)$$

$$x=0 \quad x+4=0$$

$$x=-4$$

$$x(x-4)$$

$$x=0 \quad x=4$$

Look at the graphs of the functions given above to see if you can find patterns that relate the values of a and b in the rules $y = ax^2 + bx$ to locate the features below. It may help to think about the functions using the equivalent factored form, $x(ax + b)$.

$$y = ax^2 + bx + c \quad \text{or} \quad y = ax^2 + bx \xrightarrow{\text{Factored}} y = x(ax + b)$$

$$y = x^2 + 4x$$

$$y = x(x + 4)$$

$x = 0$	$x + 4 = 0$
	$x = -4$

a. y-intercepts \rightarrow Let $x = 0$

C-value

b. x-intercepts Let $y = 0$

Use Factored Form

Set each Factor equal to zero and solve.

c. maximum or minimum point

X-value

- Factored Form
X-value is halfway between X-intercepts

• If c value $x = \frac{-b}{2a}$

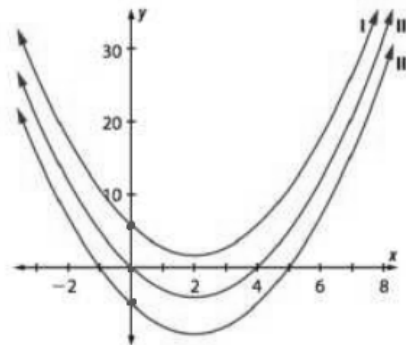
Y-value

Plug x-value back into equation

6. Explore the following examples and look for explanations of the patterns observed.

a. The diagram at the right gives graphs for three of the four quadratic functions below.

- a) $y = x^2 - 4x$ II
- b) $y = x^2 - 4x + 6$ I
- c) $y = -x^2 - 4x$
- d) $y = x^2 - 4x - 5$ III



Without using a graphing technology:

i. Determine the function with the graph that is missing on the diagram.

$$y = -x^2 - 4x \rightarrow \text{opens down}$$

ii. Match the remaining functions to their graphs, and be prepared to explain your reasoning.

b. Without using graphing technology, sketch the pattern of graphs you would expect for the next set of quadratic functions. Explain your reasoning in making the sketch.

$$y = x(x+4)$$

min (-2, -4)

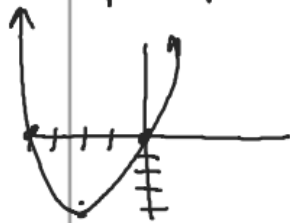
$$y = x^2 + 4x$$

Y-intercept - 0

X-intercepts

$$x = 0, -4$$

opens up



$$y = x^2 + 4x - 6$$

Y-intercept - 6

opens up

$$x = \frac{-b}{2a} = \frac{-4}{2(1)} = -2$$

$$(-2)^2 + 4(-2) - 6$$

$$4 - 8 - 6$$

$$\text{min}(-2, -10)$$

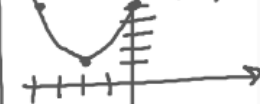
$$y = x^2 + 4x + 5$$

$$x = \frac{-4}{2(1)} = -2$$

$$y = (-2)^2 + 4(-2) + 5$$

$$4 - 8 + 5$$

$$\text{min}(-2, 1)$$



Y-intercept

5

opens up

$$x = \frac{-b}{2a}$$

c. How would the sketch showing graphs of the following functions be similar to and different from those in Part a and b? Explain your reasoning.

$$y = -x^2 + 4x$$

$$-x(x-4)$$

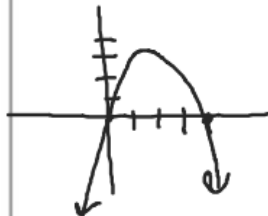
$$x=0 \quad x=4$$

$$y = -x^2 + 4x$$

$$y = -x^2 + 4x - 6$$

$$y = -x^2 + 4x + 5$$

$$\frac{-b}{2a} = \frac{-4}{2(-1)} = 2$$



d. How can the properties of the special quadratic functions $y = ax^2$, $y = ax^2 + c$, and $y = ax^2 + bx$ help in reasoning about shape and location of graphs for functions in the form $y = ax^2 + bx + c$?