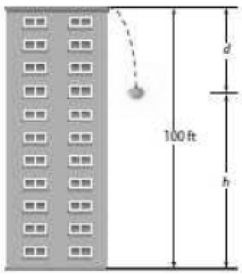
What you will learn about:
Functions that model patterns of change



Pumpkin' Droppin' At Old Dominion University in Norfolk Virginia, physics students have their own flying pumpkin contest. Each year they see who can drop pumpkins on a target from 10 stories up in a tall building while listening to music by the group Smashing Pumpkins.

By timing the flight of the falling pumpkins, the students can test scientific discoveries made by Galilei, nearly 400 years ago. Galileo used clever experiments to discover that gravity exerts force on any free-falling objects so that *d*, the distance fallen, will be related to time **t** by the function

$$d = 16t^2 \ (time\ in\ seconds\ and\ distance\ in\ feet).$$

1. Use the table to show the estimated distance the pumpkin has fallen and height above the ground in feet at various times between 0 and 3 seconds.

$16(1)^2$

$16(1.5)^2$

$16(2.25)$

Time **t**	Distance Fallen ***d***	Height Above Ground ***h***
0	0	100
0.5	4	100 - 4 = 96
1	16	100 - 16 = 84
1.5	36	100 - 36 = 64
2	64	100 - 64 - 36
2.5	100	100 - 100 = 0
3	144	100 - 144 = (-44)

2. Use the data relating height and time to answer the following questions about flight of a pumpkin dropped from a position 100 feet above the ground.

a. What function rule shows how the pumpkin's height ***h*** is related to **t**? $h(t) = 100 - 16t^2$

$h(t) = 100 - 16t^2$ ~~$16t^2\ 100$~~ $-16t^2 + 100$

$h(t) = \underline{10}$

b. What equation can be solved to find the <u>time</u> when the pumpkin is 10 feet from the ground? What is your best estimate for the solution of that equation?

$10 = 100 - 16t^2$

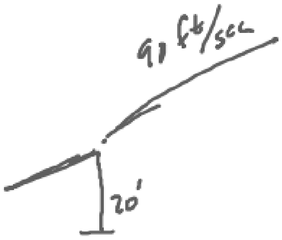
Between 2 + 2.5 Closer to 2.5

$h(t) = 0$

c. What equation can be solved to find the time when the pumpkin hits the ground? What is your best estimate for the solution of that equation?

$0 = 100 - 16t^2$

$t = 2.5$



3. Suppose a pumpkin is fired straight upward from the barrel of a compressed-air cannon at a point 20 feet above the ground, at a speed of 90 feet per second (about 60 miles per hour).

$h(x) = mx + b$

a. If there were no gravitational forces pulling the pumpkin back towards the ground, how would the pumpkin's height above the ground change as time passes?

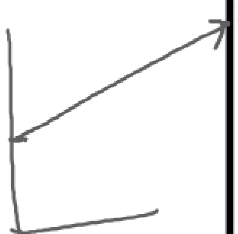
Keep going up at a rate of 90 ft/sec

$h(t) = 90h \frac{ft}{t}$

$h \cdot 90\ ft/t$

b. What function rule would relate height above the ground ***h*** in feet to time in the air ***t*** in seconds?

$h(t) = \underline{90t + 20}$

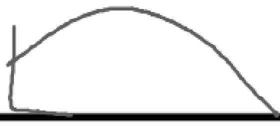


c. How would you change the function rule in part b if the pumpkin' chunker use a stronger cannon that fired the pumpkin straight up into the air with a velocity of <u>120 feet</u> per second? $h(t) = 120t + 20$

d. How would you change the function rule in Part b if the end of the cannon barrel was only 15 feet above the ground?

$h(t) = 90t + 15$

$h(t)=120t+15$



$h(t)=90t+20+90t^{-2}$

4. Now think about how the flight of a launched pumpkin results from the combination of three factors:
   - Initial height of the pumpkin's release (20 ft)
   - Initial upward velocity produced by the pumpkin-launching device, and 90 ft/sec
   - Gravity pulling the pumpkin down towards the ground.

   a. Suppose a compressed air cannon fires a pumpkin straight up into the air from a height of 20 feet and provides an initial upward velocity of 90 feet per second. What function rule would combine these conditions and the effect of gravity to give a relation between the pumpkin's height ***h*** in feet and its flight time ***t*** in seconds?

$-16t^2+90t+20$

$$h(t) = 20 + 90t \underline{-16t^2}$$

   b. How would you change your function rule in Part a if the pumpkin is launched at a height of 15 feet with an initial upward velocity 120 feet per second?

$$h(t)=15+120t-16t^2$$

5. By now you may have recognized that the height of a pumpkin shot straight up into the air at any time in its flight will be given by a function that can be expressed with a rule in the general form.

$h_0 \to h$ sub zero

h sub not

$$h = h_0 + v_0t - 16t^2$$

In those functions, ***h*** is measured in feet and ***t*** in seconds.

$V_0 \to V$ sub zero

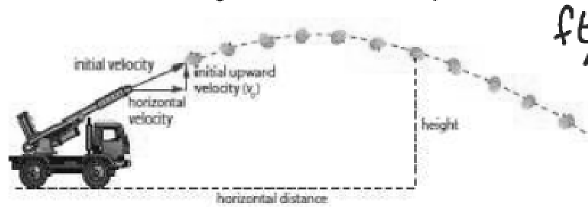
V sub not

   a. What does the value h_0 represent? What units are used to measure h_0? $h_0 \to$ Intial height

feet

   b. What does the value v_0 represent? What units are used to measure v_0? $V_0 \to$ Intial upwards velocity

ft/sec



6. The pumpkin's height in feet t seconds after it is launched will still be given by $h = h_0 + v_0 t - 16t^2$. It is fairly easy to measure the initial height (h_0) from which the pumpkin is launched, but it is not easy to measure the initial upward velocity (v_0).

a. Suppose that a pumpkin leaves a cannon at a point 24 feet above the ground when ***t*** = 0. What does that fact tell about the rule giving height ***h*** as a function of time in flight ***t***?

24 ft is the intial height

b. Suppose you were able to use a stopwatch to discover that the pumpkin shot descibed in Part a returned to the ground after 6 seconds. Use that information to find the value of v_0. 92 ft/sec

$0 = 24 + V_0(6) - 16(6)^2$

$0 = 24 + 6V_0 - 576$

$0 = 6V_0 - 552$

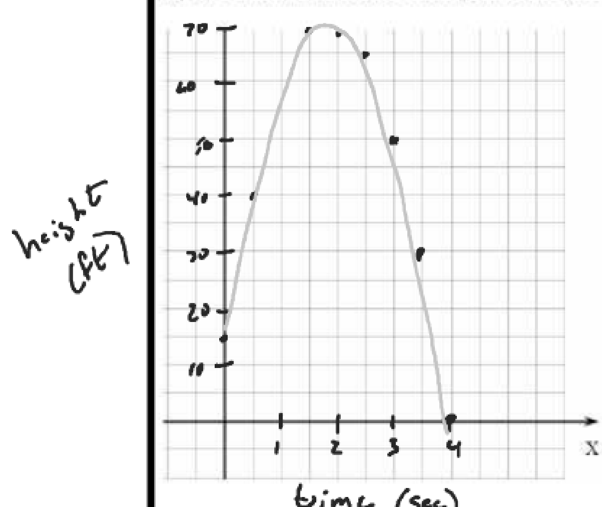
$552 = 6V_0$

$V_0 = 92$ ft/sec

7. Suppose that you were able to use a ranging tool that records the height of a flying pumpkin every half second from the time it left a cannon. A sample of the data for one pumpkin launch appears in the following table.

Time (in seconds)	0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0
Height (in feet)	15	40	60	70	70	65	50	30	0

a. Plot the data on a graph and experiment with several values of v_0 and h_0 in search of a function that models the data pattern well.



b. Use a calculator that offers quadratic curve-fitting to find a quadratic model for the sample data pattern. Compare that automatic curve-fit to what you found with your own experimentation.