

Chapter 3: Exponential, Logistic, and Logarithmic Functions
3.4: Properties of Logarithmic Functions

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What you'll Learn About

Use your Calculator to Determine which of the following are True.

1. $\log(5 + 2) \neq \log 5 + \log 2$

2. $\log(5 \cdot 2) = \log 5 + \log 2$

3. $\log(5 - 2) \neq \log 5 - \log 2$

4. $\log\left(\frac{5}{2}\right) = \log 5 - \log 2$

5. $\log(5 \cdot 2) \neq 2 \log 5$

6. $\log\left(\frac{5}{2}\right) \neq \frac{\log 5}{\log 2}$

7. $\log(5^2) \neq \log 5 \cdot \log 5$

8. $\log(5^2) = 2 \log 5$

9. $\ln(x + 2) \neq \ln x + \ln 2$

10. $\log(7x) \neq 7 \log x$

11. $\log(5x) = \log 5 + \log x$

12. $\ln\left(\frac{x}{5}\right) = \ln x - \ln 5$

13. $\log\left(\frac{x}{4}\right) \neq \frac{\log x}{\log 4}$

14. $\log_4 x^3 = 3 \log_4 x$

15. $\ln(x^2) \neq \ln x \cdot \ln x$

16. $\log|4x| = \log 4 + \log|x|$

Let b , R , and S are positive real numbers with $b \neq 1$, and c any real number

- $\log_b(RS) = \log_b R + \log_b S$

- $\log_b\left(\frac{R}{S}\right) = \log_b R - \log_b S$

- $\log_b R^c = c \log_b R$

Prove the Product Rule for Logarithms: $\log_b(RS) = \log_b R + \log_b S$

Let $x = \log_b R$ and $y = \log_b S$

$$x = \log_b R \quad y = \log_b S$$

$$b^x = R \quad b^y = S$$

$$\log_b(RS) = \log_b b^x + \log_b b^y$$

$$\log_b(b^x \cdot b^y) = x + y$$

$$\log_b(b^{x+y}) = x + y$$

$$x + y = x + y$$

Assuming x and y are positive, use properties of logarithms to write the expression as a **sum or difference** of logarithms or multiples of logarithms

A) $\log(8x)$

$$\log(8x) = \log 8 + \log x$$

B) $\ln\left(\frac{5}{x}\right)$

$$\ln\left(\frac{5}{x}\right) = \ln 5 - \ln x$$

C) $\log_2(x^5) = 5 \log_2 x$

D) $\log(8x^2y^4) = \log 8 + \log x^2 + \log y^4$
 $= \log 8 + 2 \log x + 4 \log y$

E) $\ln\left(\frac{\sqrt{x^2+5}}{\sqrt[3]{x^4}}\right) = \ln\left(\frac{(x^2+5)^{1/2}}{(x^{4/3})}\right)$
 $= \ln((x^2+5)^{1/2}) - \ln(x^{4/3})$
 $= \frac{1}{2} \ln(x^2+5) - \frac{4}{3} \ln x$

Let b , R , and S are positive real numbers with $b \neq 1$, and c any real number

- $\log_b(RS) = \log_b R + \log_b S$
- $\log_b\left(\frac{R}{S}\right) = \log_b R - \log_b S$
- $\log_b R^c = c \log_b R$

Assuming x , y and z are positive, use properties of logarithms to write the expression as a single logarithm

A) $\log x + \log 6 = \log(6x)$ B) $\ln x - \ln 6 = \ln\left(\frac{x}{6}\right)$

C) $\frac{1}{4} \log x = \log x^{1/4}$
 $= \log \sqrt[4]{x}$

D) $6 \log x - \frac{1}{2} \log y$
 $\log x^6 - \log y^{1/2}$
 $\log\left(\frac{x^6}{\sqrt{y}}\right)$

E) $5 \log(x^2 y) + 3 \log(y^2 z)$

$\log(x^2 y)^5 + \log(y^2 z)^3$
 $\log(x^{10} y^5) + \log(y^6 z^3) = \log(x^{10} y^5 \cdot y^6 z^3)$
 $= \log(x^{10} y^{11} z^3)$

F) $\ln x^5 - 2 \ln(xy)$

$\ln x^5 - \ln x^2 y^2$
 $\ln\left(\frac{x^5}{x^2 y^2}\right) = \ln\left(\frac{x^3}{y^2}\right)$

2.3.2.3

$(xy)^2$

$(xy)(xy)$

$x^2 \cdot y^2$