

Bacteria Growth

The number of bacteria after t hours is given by

$$y = 150 e^{0.521t}$$

a) What was the initial amount of bacteria present?

$$t = 0 \quad y = 150 e^{0.521(0)} \quad y = 150$$

b) How many bacteria are present after 4 hours?

$$y = 150 e^{0.521(4)}$$

c) How many hours will it take until there are 400 bacteria?

$$400 = 150 e^{0.521t}$$

↑ Plug into y_2 ↑ Plug into y_1

Find the intersection

2nd Trace 5

$$\frac{400}{150} = \frac{150 e^{0.521t}}{150}$$

$$\frac{400}{150} = e^{0.521t}$$

$$\ln\left(\frac{400}{150}\right) = \ln e^{0.521t}$$

$$\frac{\ln\left(\frac{400}{150}\right)}{0.521} = \frac{0.521t}{0.521}$$

$$\frac{\ln\left(\frac{400}{150}\right)}{0.521} = t$$

$$\ln e^{0.521t}$$

$$0.521t (\ln e)$$

$$0.521t (1)$$

$$0.521t$$

Determine a formula for the exponential function whose values are given
20)

x	g(x)
-2	-9.0625 ✓
-1	-7.25 ✓
0	-5.8
1	-4.64 ✓
2	-3.7123 ✓

Initial Value →

$$y = a \cdot b^x$$

$$y = -5.8 b^x$$

$$\frac{-4.64}{-5.8} = \frac{-5.8 b^1}{-5.8}$$

$$b = .8$$

$$y = -5.8 (.8)^x$$

Determine a formula for the exponential function whose points are given

21) (0, 4) (5, 8.05)

Find the logistic function that satisfies the given conditions

A) Initial Value 6: Max Capacity (Limit to growth) = 30
Passing through (1, 15)

$$y = \frac{M}{1 + Ab^x}$$

$$y = \frac{30}{1 + Ab^x}$$

$$(0, 6)$$

$$6 = \frac{30}{1 + Ab^0}$$

$$(1, 15)$$

$$15 = \frac{30}{1 + 4b^1}$$

$$y = \frac{30}{1 + 4\left(\frac{1}{4}\right)^x}$$

$$6 = \frac{30}{1 + A}$$

$$1 + A = \frac{30}{6}$$

$$1 + 4b = \frac{30}{15}$$

$$1 + 4b = 2$$

$$4b = 1$$

$$1 + A = 5$$

$$b = \frac{1}{4}$$

$$A = 4$$

$$\left(\frac{1}{12}\right)^{1/4} = b = \sqrt[4]{1/12}$$

A: Initial Value based on the Max Capacity

$$y = \frac{100}{1 + 4\left(\frac{1}{12}\right)^{1/4 x}}$$

B) Initial Population = 20, Max Capacity (Limit to growth) = 100
Passing through (4, 75)

$$y = \frac{100}{1 + A \cdot b^x}$$

$$20 = \frac{100}{1 + A b^0}$$

$$75 = \frac{100}{1 + 4b^4}$$

$$20 = \frac{100}{1 + A}$$

$$1 + 4b^4 = \frac{100}{75}$$

$$1 + A = \frac{100}{20}$$

$$1 + 4b^4 = 4/3$$

$$1 + A = 5$$

$$A = 4$$

$$\frac{4b^4}{4} = \frac{(1/3)}{4}$$

$$b^4 = 1/12$$

32. Exponential Growth: The population of River City in the year 1910 was 4200. Assume the population increased at a rate of 2.25% per year.

1910 → 4200
1930 →

a) Estimate the population in 1930.

$$y = a b^x$$

$$y = 4200(1.0225)^x$$

$$y = 4200(1.0225)^{20}$$

b) Predict when the population reached 20,000.

$$\frac{20000}{4200} = \frac{4200(1.0225)^x}{4200}$$

$$\ln\left(\frac{20000}{4200}\right) = \ln(1.0225)^x$$

$$\ln\left(\frac{20000}{4200}\right) = x \ln(1.0225)$$

$$\frac{20000}{4200} = 1.0225^x$$

$$\frac{\ln(20000/4200)}{\ln(1.0225)} = x$$

Example 4: Suppose the half-life of a certain radioactive substance is 20 days and there are 5 grams present initially.

A) Express the amount of the substance remaining as function of time.

B) Find the time when there will be 1 gram of the substance remaining.

Watauga High School has 1200 students. Bob, Carol, Ted and Alice start a rumor, which spreads logistically so that

$S(t) = \frac{1200}{1 + 39e^{-0.9t}}$ models the number of students who have heard the rumor by the end of day t .

A) How many students have heard the rumor by the end of Day 0.

$$S(0) = \frac{1200}{1 + 39e^{-0.9(0)}} = \frac{1200}{1 + 39} = \frac{1200}{40} = 30$$

B) How long does it take for 1000 students to hear the rumor?

$$1000 = \frac{1200}{1 + 39e^{-0.9t}}$$

$$\frac{39e^{-0.9t}}{39} = \frac{2}{39}$$

$$e^{-0.9t} = \frac{2}{39}$$

$$\ln e^{-0.9t} = \ln\left(\frac{2}{39}\right)$$

$$-0.9t = \ln\left(\frac{2}{39}\right)$$

$$1 + 39e^{-0.9t} = \frac{1200}{1000}$$

$$1 + 39e^{-0.9t} = 1.2$$

$$t = \frac{\ln\left(\frac{2}{39}\right)}{-0.9}$$

Use the data in the table and exponential regression to predict Dallas, TX population in 2015.

1950	434,462
1960	679,684
1970	844,401
1980	904,599
1990	1,006,877
2000	1,188,589