

CALCULUS: Graphical, Numerical, Algebraic by Finney, Demana, Waits and Kennedy
Chapter 9: Convergence of Series

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What you'll Learn About
Interval and Radius of Convergence

Find the Radius of Convergence

$$8) \sum_{n=0}^{\infty} (x+5)^n = \quad r = |x+5|$$

$$\begin{array}{c} -1 < x+5 < 1 \\ -5 \qquad -5 \qquad -5 \\ \hline -6 < x < -4 \end{array}$$

$$R.O.C = 1$$

$$8) \sum_{n=0}^{\infty} (x+5)^n =$$

$$\lim_{n \rightarrow \infty} \left| \frac{(x+5)^{n+1}}{(x+5)^n} \right| = |x+5| < 1$$

$$\frac{(x+5)^n \cdot (x+5)^1}{(x+5)^n}$$

$$\begin{array}{c} -1 < x+5 < 1 \\ -5 \qquad -5 \qquad -5 \\ \hline -6 < x < -4 \end{array}$$

$$R.O.C = 1$$

$$12) \sum_{n=0}^{\infty} \frac{nx^n}{n+2} =$$

$$\lim_{n \rightarrow \infty} \left| \frac{(n+1)x^{n+1}}{(n+3)} \cdot \frac{(n+2)}{n x^n} \right| = \left| \frac{x(n+1)(n+2)}{n(n+3)} \right| = |x| \cdot 1$$

I.O.C. $-1 < x < 1$

R.O.C. = 1

$$18) \sum_{n=0}^{\infty} \frac{\sqrt{n}x^n}{3^n} =$$

$$\lim_{n \rightarrow \infty} \left| \frac{\sqrt{n+1} x^{n+1}}{3^{n+1}} \cdot \frac{3^n}{\sqrt{n} x^n} \right| = \left| \frac{x \sqrt{n+1}}{3 \sqrt{n}} \right| = \left| \frac{x}{3} \right| \cdot 1$$

$-1 < \frac{x}{3} < 1$

I.O.C. $-3 < x <$

R.O.C. = 3

$$A) \sum_{n=0}^{\infty} \frac{(-1)^n x^n}{(2n)!} =$$

$$\lim_{n \rightarrow \infty} \left| \frac{(-1)^{n+1} x^{n+1}}{(2n+2)!} \cdot \frac{(2n)!}{(-1)^n x^n} \right| = \left| \frac{x}{(2n+2)(2n+1)} \right| = 0 < 1$$

Absolutely Converges Always

I.O.C. $-\infty < x < \infty$

R.O.C. = ∞

$$15) \sum_{n=1}^{\infty} (n+1)! x^n =$$

$$\lim_{n \rightarrow \infty} \left| \frac{(n+2)! x^{n+1}}{(n+1)! x^n} \right| = \left| x(n+2) \right| = \infty > 1$$

$$\frac{(n+2)!}{(n+1)!} = n+2$$

diverges, except
at $x = 0$

$$\sum_0^{\infty} 0 = 0 + 0 + 0 + 0$$

\uparrow
(center)