CALCULUS: Graphical, Numerical, Algebraic by Finney, Demana, Watts and Kennedy Chapter 9: Error and Series 9.3:

What you'll Learn About How to find the error of a series that does not alternate

Lagrange Error Bound/Taylors Inequality/Remainder Estimation Theorem

- Give the first term of the series for $f(x) = e^x$ centered at x = 0 $P_{i}(x) = 1$
- Find the approximation for P(.1) = 4
- Find $f(.1) = e^{-1} = 1.105170918$
- How accurate is the approximation. $|f(x) P_1(x)| = .105170918$
- What is the value of the next term of the polynomial at x = .1

Give the first two terms of the series for $f(x) = e^x$ centered at x = 0

- Find the approximation for P(.1) = [-1, 1] = [-1, 1]
- Find f(.1) = [105170918
- How accurate is the approximation. $|f(.1) P_1(.1)| = .005170918$
- What is the value of the next term of the polynomial at x = .1

Next term =
$$\frac{x^2}{2!} = .005$$

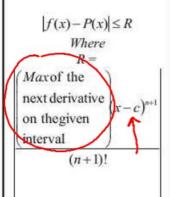
Give the first three terms of the series for $f(x) = e^x$ centered at x = 0

- $1 + x + \frac{x^2}{2}$ Find the approximation for P(.1) = $1 + \frac{1}{2} = 1.105$
- Find f(.1) = 1.105170918

13/0)=1

- How accurate is the approximation. $|f(x) \rho_2(x)| = .000170918$
- What is the value of the next term of the polynomial at x = .1

$$\frac{25|\text{Page}(\frac{30}{3})x^{3}|}{3!} = \text{next term} = \frac{1}{3!} \rightarrow \frac{1^{3}}{6} = \frac{.001}{6} = .000166666$$



Where x-c is the distance from the center

Where n is the order

We must build the next term a little bit bigger to have a good boundary for the error.

Remember, whenever you see this, $|f(x)-P(x)| \le R$, you are finding error bound

whenever you see this, f(x) - P(x), you are finding the actual error between the function and the approximation from the polynomial

Actual difference

Use Taylors Inequality to determine the error bound $|f(x)-P(x)| \le R$

Give the first 4 terms of the series for $f(x)=e^x$ centered at x=0

$$f'(x) = e^{x}$$

$$f'(x) = e^{x}$$

$$f''(x) = e^{x}$$

$$f''(x)$$

$$f(x) = \frac{1}{(1-x)^2} = (1-x)^{-2} \qquad f(0) = 1$$

$$f'(x) = +2(1-x)^{-3} \qquad f'(0) = 0$$

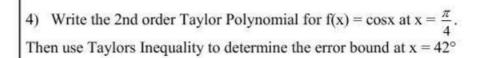
$$f''(x) = +6(1-x)^{-4} \qquad f''(0) = 0$$

$$f'''(x) = 24(1-x)^{-5} \qquad f^{3}(0) = 24$$

$$P_3(x) = 1 + 2x + 6x^2 + 24x^3$$

Find the Lagrange error bound f(x)-P(x) R for the series between $0 \le x \le .2$ Build next form

$$\frac{120(.8)^{-6}x^{4}}{4!} = \frac{120}{(.8)^{6}}x^{4}$$



5) Write the 1st degree Taylor Polynomial for $f(x) = \arcsin x$ at x = 0. Then use Taylors Inequality to determine the error bound at x = .2

Nex+ Term
$$f'(x) = \frac{1}{\sqrt{1-x^2}} = (1-x^2) f'(0) = 1$$

$$f'(x) = \frac$$

from center on both sides of center

9) Given that $P_1(x) = x$ represents the first order polynomial for sinx centered at x = 0. Use the Lagrange Error Bound to find the error when $|x| \le .05$

Build next term

ti(1) = cosx

$$x = -.05$$

$$x = 0$$

$$x = -.05$$

$$x = 0$$

$$x = .05$$

$$x = .0$$

14) Given that $P_3(x) = (x-1) - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3$ represents the third order Taylor polynomial for $\ln(x)$ centered at x = 1. Use the Lagrange Error Bound to find the error when $|x-1| \le .1$

Summary of Error Bound

For an Alternating Series - Use the next term

For a series that is Not Alternating

- 1. Write down the formula for the next derivative.
- 2. Find the value of the next derivative at the ends of the interval and the center.
- 3. Whichever value is bigger is the value you use to build your error bound term

