12) a) Find all values of x for which the geometric series $\sum_{n=0}^{\infty} e^{nx}$ converges.

$r = e^x$

$-1 < e^x < 1$

$\ln(-1) < x < \ln 1$

$\ln(-1) < x$

$-1 < e^x$

always

$x < \ln 1$

$\boxed{x < 0}$

b) Find the function (sum) represented by the series $\sum_{n=0}^{\infty} e^{nx}$

$f(x) = \dfrac{1}{1-e^x}$

c) Find all values of x for which $\sum_{n=0}^{\infty} e^{nx} > 2$

sum

$\dfrac{1}{1-e^x} > 2$

$1 > 2(1-e^x)$

$1 > 2 - 2e^x$

$-1 > -2e^x$

$\frac{1}{2} < e^x$

$\boxed{\ln\frac{1}{2} < x}$

d) Find all values of x for which $\sum_{n=0}^{\infty} e^{nx} < 1$

$\dfrac{1}{1-e^x} < 1$

$1 < 1 - e^x$

$0 < -e^x$

$0 > e^x$

$\ln(0) < x$

None

Not possible

13) Let $f(x) = \sum_{n=0}^{\infty} \frac{(-2)^n x^n}{3^n(n+1)^2}$

(a) Find the interval of convergence of the series.

$$\lim_{n\to\infty}\left|\frac{(-2)^{n+1}x^{n+1}}{(3)^{n+1}(n+2)^2}\cdot\frac{3^n(n+1)^2}{(-2)^n x^n}\right| = \left|\frac{2x(n+1)^2}{3(n+2)^2}\right| = \boxed{\left|\frac{2x}{3}\right| < 1}$$

(b) For what values of x does the series converge absolutely?

$$-1 < \frac{2x}{3} < 1$$

$$-3 < 2x < 3$$

$$\boxed{-\frac{3}{2} \le x \le \frac{3}{2}}$$

$x = -\frac{3}{2}$

$$\sum \frac{(-2)^n\left(-\frac{3}{2}\right)^n}{3^n(n+1)^2}$$

$$\sum \frac{1}{(n+1)^2}$$ compare to $\sum \frac{1}{n^2}$

$\frac{1}{(n+1)^2} < \frac{1}{n^2}$ OR $\lim_{n\to\infty} \frac{\frac{1}{(n+1)^2}}{\frac{1}{n^2}} = 1$ $p=2>1$ converges

$x = \frac{3}{2}$ $\sum \frac{(-2)^n\left(\frac{3}{2}\right)^n}{3^n(n+1)^2}$

$\sum \frac{(-1)^n}{(n+1)^2}$

$\lim_{n\to\infty} \frac{1}{(n+1)^2} = 0$ $\frac{1}{(n+2)^2} < \frac{1}{(n+1)^2}$

Absolute

$\sum \frac{1}{(n+1)^2}$

(c) Find the domain (interval of convergence) of the following function: $h(x) = f(x^2)$.

$$\sum \frac{(-2)^n x^{2n}}{3^n(n+1)^2}$$

$$-1 < \frac{2x^2}{3} < 1$$

$$-3 < 2x^2 < 3$$

$$-\frac{3}{2} < x^2 < \frac{3}{2}$$

$$\boxed{-\sqrt{\frac{3}{2}} \le x \le \sqrt{\frac{3}{2}}}$$

$$\lim_{n\to\infty}\left|\frac{(-2)^{n+1}x^{2n+2}}{3^{n+1}(n+2)^2}\cdot\frac{3^n(n+1)^2}{(-2)^n x^{2n}}\right| = \left|\frac{2x^2}{3}\right| < 1$$

$x = -\sqrt{\frac{3}{2}}$

$$\sum \frac{(-2)^n\left(\frac{3}{2}\right)^n}{3^n(n+1)^2} = \sum \frac{(-1)^n}{(n+1)^2}$$

$X = \sqrt{\frac{3}{2}}$

$$\sum \frac{(-2)^n\left(\frac{3}{2}\right)^n}{3^n(n+1)^2} \xrightarrow{\text{Same}}$$

By the alt series in part b, the endpoint has absolute convergence