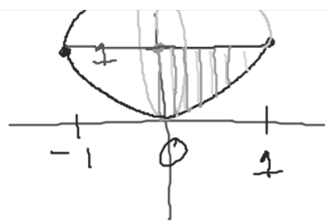
p. 407 31a



Find the volume of the solid generated by revolving the region bounded by $y = x^2$ and the lines y = 1 about the line y = 1

$$V = 2\pi \int_0^1 (1 - x^2)^2$$

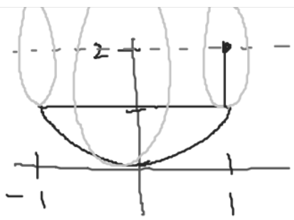
$$V = 2\pi \int_0^1 1 - 2x^2 + x^4$$

$$2\pi \left[x - \frac{2}{3}x^3 + \frac{1}{5}x^5 \right]_0^1$$

$$2\pi \left[1 - \frac{2}{3} + \frac{1}{5} \right]$$

$$2\pi \left[\frac{15}{15} - \frac{10}{15} + \frac{3}{15} \right] = \frac{16\pi}{15}$$

p. 407 31b



$\frac{60}{15} - \frac{20}{15} + \frac{3}{15}$

Find the volume of the solid generated by revolving the region bounded by $y = x^2$ and the lines y = 1 about the line y = 2

$56\pi/15$

$V = 2\pi \int_0^1 (2-x^2)^2 - 2\pi \int_0^1 (2-1)^2$

$\frac{86\pi}{15} - \frac{30\pi}{15}$

$V = 2\pi \int_0^1 4 - 4x^2 + x^4 - 2\pi \int_0^1 1$

$2\pi \left(\frac{43}{15}\right) - 2\pi$

$V = 2\pi \left[4x - \frac{4}{3}x^3 + \frac{1}{5}x^5 \right]_0^1 - 2\pi \left[x \right]_0^1 = 2\pi \left[4 - \frac{4}{3} + \frac{1}{5} \right] - 2\pi$

p. 407 31c

Find the volume of the solid generated by revolving the region bounded by $y = x^2$ and the lines y = 1 about the line y = -1

$$V = 2\pi \int_0^1 \left(1-(-1)\right)^2 - 2\pi \int_0^1 \left(x^2-(-1)\right)$$

$$V = 2\pi \int_0^1 2^2 - 2\pi \int_0^1 x^4 + 2x^2 + 1$$

$$V = 2\pi \left[4x\right]_0^1 - 2\pi \left[\frac{1}{5}x^5 + \frac{2}{3}x^3 + x\right]_0^1$$

$$8\pi - 2\pi\left[\frac{1}{5} + \frac{2}{3} + 1\right]$$

$$8\pi - 2\pi\left[\frac{3}{15} + \frac{10}{15} + \frac{15}{15}\right]$$

$$8\pi - 2\pi\left[\frac{28}{15}\right]$$

$$8\pi - \frac{56\pi}{15}$$

$$\frac{120\pi}{15} - \frac{56\pi}{15}$$

$$\frac{64\pi}{15}$$

$$\begin{array}{r} 15 \\ \times\ 8 \\ \hline 120 \end{array} \qquad \begin{array}{r} 120 \\ -56 \\ \hline 64 \end{array}$$

p. 408 39b

length of each side of square

The base of a solid is the region between the curve $y = 2\sqrt{\sin x}$ and the interval $[0, \pi]$ and the x-axis. The cross sections perpendicular to the x-axis are squares with bases running from the x-axis to the curve. Find the volume of the solid.

$$V = \int_0^{\pi} \left(2\sqrt{\sin x}\right)^2$$

$$V = \int_0^{\pi} 4\sin x = -4\cos x\Big]_0^{\pi} = -4\cos\pi - \left(-4\cos(0)\right)$$

$$= -4(-1) + 4(+1)$$

$$4 + 4 \quad 8$$

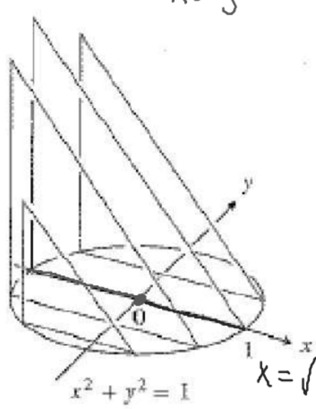
p. 408 42

$x^2+y^2=1$

$x^2=1-y^2 \qquad x=\pm\sqrt{1-y^2}$

The base of a solid is the disk $x^2+y^2=1$. The cross sections by planes perpendicular to the y-axis between y = -1 and y = 1 are isosceles triangles with one leg in the disk.

$2\sqrt{1-y^2}$ = base
height



$x=\sqrt{1-y^2}$

$$V=\frac{1}{2}\int_{-1}^{1}\left(2\sqrt{1-y^2}\right)\left(2\sqrt{1-y^2}\right)$$

$$V=2\int_{-1}^{1}1-y^2$$

$$V=2\left[y-\frac{1}{3}y^3\right]_{-1}^{1}=2\left[\left(1-\frac{1}{3}\right)-\left(-1+\frac{1}{3}\right)\right]$$

$$2\left[\frac{2}{3}+\frac{2}{3}\right]=\frac{8}{3}$$

p. 416 #8 Find the length of the curve

$$x = \int_0^y \sqrt{\sec^2 t - 1} \qquad \frac{-\pi}{3} \le y \le \frac{\pi}{4}$$

2.198

$$\frac{dx}{dy} = \sqrt{\sec^2 y - 1} \qquad L = \int \sqrt{1 + \left(\frac{dx}{dy}\right)^2}$$

$$L = \int_{-\frac{\pi}{3}}^{\pi/4} \sqrt{1 + (\sec^2 y - 1)} = \int_{-\frac{\pi}{3}}^{\pi/4} \sqrt{\sec^2 y} = \int_{-\frac{\pi}{3}}^{\pi/4} \sec y$$

p. 416 #11 Find the length of the curve

$$y=\frac{1}{3}\left(x^2+2\right)^{3/2} \qquad x=0 \text{ to } x=3$$

$$\frac{dy}{dx}=\frac{1}{2}\left(x^2+2\right)^{1/2}\cdot 2x$$

$$\frac{dy}{dx}=x\sqrt{x^2+2}$$

$$\left(\frac{dy}{dx}\right)^2=x^2\left(x^2+2\right)=x^4+2x^2$$

$$L=\int_0^3\sqrt{1+x^4+2x^2}$$

$$L=\int_0^3\sqrt{x^4+2x^2+1}=12$$

$$=\int_0^3\sqrt{\left(x^2+1\right)^2}=\int_0^3 x^2+1$$

$$=\frac{1}{3}x^3+x\Big]^3$$