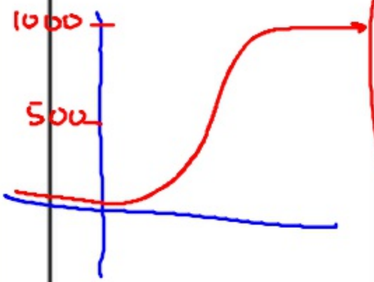
In 1985 and 1987, the Michigan Department of Natural Resources airlifted 61 moose form Algonquin Park, Ontario to Marquette County in the Upper Peninsula. It was originally hoped that the population P would reach carrying capacity in about 25 years with a growth rate of

$$\frac{dp}{dt} = .0003P(1000 - P) = .3P - .0003P^2$$

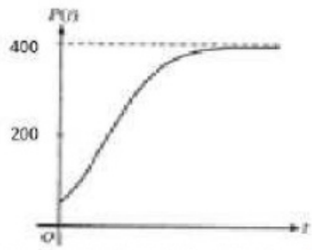
Solve the differential equation with the initial condition P(0) = 61.



$$\boxed{\frac{dP}{dt} = .0003P(1000-P)} \rightarrow P = \frac{1000}{15.393e^{-.3t}+1}$$

$\lim_{t \to \infty} P(t) =$ Max Capacity

Point of Inflection $= \frac{1}{2}$ of Max Capacity



$.0008P(25 - P)$

24. Which of the following differential equations for a population P could model the logistic growth shown in the figure above?

A) $\frac{dP}{dt} = .02P - 0.0008P^2$ B) $\frac{dP}{dt} = .08P - .0002P^2$

C) $\frac{dP}{dt} = .8P^2 - 0.0002$ D) $\frac{dP}{dt} = 0.08P^2 - .0002$

E) $\frac{dP}{dt} = 0.08P^2 - 0.0002P$

$.0002P(400 - P)$

21. The number of moose in a national park is modeled by the function M that satisfies the logistic differential equation $\frac{dM}{dt} = .05M\left(1 - \frac{M}{1000}\right)$, where t is the time in years and M(0)=50. What is the $\lim_{t\to\infty} M(t)$?

A) 50 B) 200 C) 500 D) 1000 E) 2000

$\frac{1}{\left(\frac{1}{1000}\right)} =$

$\frac{dm}{dt} = .05m\left(1 - \frac{m}{1000}\right)$

$= .05m \frac{1}{1000}(1000 - m)$

P

$\frac{dP}{dt}$

84. The rate of change, $\frac{dP}{dt}$, of the number of people on an ocean beach is modeled by a logistic differential equation. The maximum number of people allowed on the beach is 1000. At 10 A.M., the number of people on the beach is 400 and is increasing at the rate of 200 people per hour. Which of the following differential equations describes the situation.

A) $\frac{dP}{dt} = \frac{1}{200}(1000 - P)$

B) $\frac{dP}{dt} = \frac{1}{2}P(1000 - P) + 100$

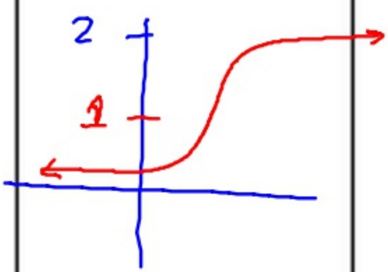
C) $\frac{dP}{dt} = \frac{1}{3}(1000 - P)$

D) $\frac{dP}{dt} = \frac{1}{1200}P(1000 - P)$

E) $\frac{dP}{dt} = 200P(1000 - P)$

26. The population P(t) of a species satisfies the logistic differential equation $\frac{dP}{dt} = P\left(4 - \frac{P}{2000}\right)$, where the initial position P(0)=1500 and t is the time in years. What is $\lim_{t \to \infty} P(t)$?

A) 2500 B) 8000 C) 4200 D) 2000 E) 4000



Let g be a function with g(4) = 1, such that all points (x, y) on the graph of g satisfy the logistic differential equation $\frac{dy}{dx} = 3y(2 - y)$.

b) Given that g(4) = 1, find $\lim_{x \to \infty} g(x)$ and $\lim_{x \to \infty} g'(x)$.

$\lim_{x \to \infty} g(x) = 2$ $\lim_{x \to \infty} g'(x) = 0$

c) For what value of y does the graph of g have a point of inflection? Find the slope of the graph of g at the point of inflection. (It is not necessary to solve for g(x).)

$y = 1$ $\frac{dy}{dx} = 3(1)(2-1) = 3$

	A population is modeled by a function P that satisfies the logistic differential equation $\frac{dP}{dt} = \frac{P}{10}\left(1 - \frac{P}{15}\right)$. a) If P(0) = 3, what is the $\lim_{t\to\infty} P(t)$? = 15 If P(0) = 20, what is the $\lim_{t\to\infty} P(t)$? = 15 b) If P(0) = 3, for what value of P is the population growing the fastest? P = 7.5 Rogawski 8. If y(x) is a solution to $\frac{dy}{dx} = 3y(10 - y)$ with y(0) = 3 then as $x \to \infty$, A) y(x) increases to ∞ B) y(x) increases to 5 C) y(x) decreases to 5 D) y(x) increases to 10 E) y(x) decreases to 10 9. If y(x) is a solution to $\frac{dy}{dx} = 4y(12 - y)$ with y(0) = 10 then as $x \to \infty$, A) y(x) decreases to ∞ B) y(x) increases to 6 C) y(x) increases to 12 D) y(x) decreases to 12 E) y(x) decreases to 0 16. If $\frac{dy}{dt} = 3y(10 - 2y)$ with y(0) = 1 then, y is increasing the fastest when A) y = 1.5 B) y = 2.5 C) y = 3 D) y = 4 E) y = 5 18. If $\frac{dy}{dt} = 3y(10 - 2y)$ with y(0) = 1, then the maximum value of y is A) y = 1 B) y = 2.5 C) y = 5 D) y = 10 E) Never attained; has no maximum value

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25. Given the differential equation $\frac{dz}{dt} = z\left(6 - \frac{z}{50}\right)$, where z(0) = 50, what is the $\lim_{t \to \infty} z(t)$?

A) 50 B) 100 C) 300 D) 6 E) 200

25. Given the differential equation $\frac{dz}{dt} = z\left(6 - \frac{z}{50}\right)$, where z(0) = 50, then z is increasing the fastest when z =

A) 150 B) 100 C) 300 D) 50 E) 100

Other Rate type problems

Rogawski

11. The rate at which a certain disease spreads is proportional to the quotient of the percentage of the population with the disease and the percentage of the population that does not have the disease. If the constant of proportionality is .03, and y is the percent of people with the disease, then which of the following equations gives R(t), the rate at which the disease is spreading.

mult by .03

A) R(t) = .03y

B) $R(t) = \frac{.03dy}{dt}$

C) $\frac{dr}{dt} = \frac{.03R}{(1-R)}$

D) $R(t) = .03\frac{y}{(1 - y)}$

have disease

don't have disease

E) $\frac{dr}{dt} = .03R$

multiplying

12. The rate of change of the volume, V, of water in a tank with respect to time, t is directly proportional to the square root of the time, t, it takes to empty the tank . Which of the following is a differential equation that describes this relationship.

~~A)~~ $V(t)=k\sqrt{t}$ ~~B)~~ $V(t)=k\sqrt{V}$ C) $\frac{dV}{dt}=k\sqrt{t}$

D) $\frac{dV}{dt}=\frac{k}{\sqrt{V}}$ E) $\frac{dV}{dt}=k\sqrt{V}$

16. Let P(t) represent the number of wolves in a population at time t years, when $t \geq 0$. The population P(t) is increasing at rate directly proportional to 500 divided by P(t), where the constant of proportionality is k. Write the differential equation that describes this relationship.

$$\frac{dP}{dt} = k\left(\frac{500}{P(t)}\right)$$

Quadratic

23. If P(t) is the size of a population at time t, which of the following differential equations describes exponential growth in the size of the population.

Linear A) $\frac{dP}{dt}=200$ B) $\frac{dP}{dt}=200t$ C) $\frac{dP}{dt}=100t^2$ cubic

D) $\frac{dP}{dt}=200P$ E) $\frac{dP}{dt}=100P^2$

$$\int \frac{dP}{P} = \int 200\,dt$$

$$\ln P = 200t + C$$

$$P = e^{200t+C}$$

$$\frac{dP}{P^2} = 100\,dt$$

$$\int P^{-2}\,dP = 100t + C$$

$$-P^{-1} = 100t + C$$

$$-\frac{1}{P} = 100t + C$$

Inverse Variation