What you'll Learn About

- How to integrate a product by recognizing that one of the pieces contains the derivative of the other

18) $\int x\cos\left(2x^2\right)dx \quad \mathrm{u} = 2\mathrm{x}^2$

21) $\int \frac{dx}{x^2+9} \qquad \mathrm{u} = \frac{x}{3}$

24) $\int 8\left(x^4+4x^2+1\right)^2\left(x^3+2x\right)dx \qquad \mathrm{u} = x^4+4x^2+1$

A) $\int (x+2)^5 dx$

B) $\int \sqrt{4x-1}dx$

C) $\int 2x\sqrt{1+x^2}dx$

D) $\int 3x^2(x^3+1)^{1/3} dx$

D) $\int (2-3t^2)\sqrt{2t^3-4t+8}dx$

E) $\int \frac{(5\sqrt{x}+2)^2}{\sqrt{x}} dx$

	$D)\quad \int \cos(7x+5)dx$ $E)\quad \int x^2 \sin(x)^3 dx$ $F)\quad \int \sin^4 x \cos x dx$

	$G)$ $\int \tan x \sec^2 x dx$ 58) $\int_{-\pi}^{\pi} \frac{\cos x}{\sqrt{4+3\sin x}} dx$ $J)$ $\int \frac{1}{x^2+81} dx$ $K)$ $\int \frac{1}{x^2+16} dx$

	$I)\quad \int \frac{2x}{x^2+81}\,dx$ $L)\quad \int \frac{4x}{x^2+20}\,dx$ $M)\quad \int \frac{4x}{\left(x^2+20\right)^3}\,dx$ $\qquad N)\quad \int \frac{\ln^9 x}{x}\,dx$

	64) $\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \cot x dx$ 66) $\int_0^2 \frac{e^x}{3+e^x} dx$

	$\int \text{Trig Function}$ - Take the antiderivative of the trig and then check $\int (\text{Polynomial})(\text{Trig Function})$ - Take the antiderivative of the trig and then check $\int (\text{Trig Function})(\text{Trig Function})$ - Take the antiderivative of one of the trig functions using the power rule and then check Unless $\int (\sec x)(\tan x)$ or $\int (\csc x)(\cot x)$ - these fall into the first category above $\int \frac{\text{trig function}}{\text{trig function whose power is not 1}}$ - Flip the denominator up to the top and then bump the power up on that denominator and check $\int \frac{\text{constant}}{x^2 + a^2} \rightarrow \frac{1}{a}\arctan\left(\frac{x}{a}\right)$ $\int \frac{\text{trig function}}{\text{polynomial function that is the derivative of the angle}}$ - Take the antiderivative of the numerator and check

	$\int(\text{Polynomial Function})(\text{Polynomial Function})$ - Take the antiderivative of one of the polynomial functions using the power rule and then check $\int \frac{\text{numerator is derivative of the bottom}}{\text{polynomial function whose power is not 1}}$ - Flip the denominator up to the top and then bump the power up on that denominator and check $\int \frac{\text{polynomial function}}{\text{polynomial function that is the derivative of part of the top}}$ - Take the antiderivative of the numerator and check $\int \frac{\text{function that is the derivative of the bottom}}{\text{function whose quantity is to the first power}}$ - Write down the ln(denominator) and check If the denominator's quantity is to the first, the antiderivative is either arctangent or natural log (ln)

CALCULUS: Graphical, Numerical, Algebraic by Finney, Demana, Watts and Kennedy
Chapter 6: Differential Equations 6.3: Tabular Integration

What you'll Learn About

- How to integrate a product by that cannot be done by recognition

Proof of Integration by Parts

1. $Find\ \frac{d}{dx}(uv) =$

2. Integrate both sides

3. Solve for $\int udv$

Use ultra violet minus super vdu to integrate the following

. 2. $\int xe^x =$

Use tabular integration to integrate the following

2. $\int xe^x =$

	Use tabular integration to integrate the following 6. $\int x^2 e^{-x} =$ 8. $\int x^2 \cos\left(\frac{x}{2}\right) =$

Solve the initial value problem using tabular integration

11. $\frac{\mathrm{dy}}{\mathrm{dx}} = (x+2)\sin x \quad \mathrm{y} = 2 \text{ and } \mathrm{x} = 0$

16. $\frac{\mathrm{dy}}{\mathrm{dx}} = 2x\sqrt{x+2} \quad \mathrm{y}(-1) = 0$

	Use tabular integration to integrate the following 10. $\int x^2 \ln x dx$ Use ultra violet minus super vdu to integrate the following 10. $\int x^2 \ln x dx$

	Use tabular integration to integrate the following $A.\ \int \arcsin(x)\,dx$ 19. $\int e^{x} \cos(2x)\,dx$

Top Heavy Integrals

$$A.\ \int \frac{x^2+x}{x}\,dx$$

$$B.\ \int \frac{\sqrt{x}+5}{x}\,dx$$

$$C.\ \int \frac{x^3+2x}{\sqrt{x}}\,dx$$

What you'll Learn About

- How integrate a fraction when the denominator can be factored and the numerator is not the derivative of the denominator

$$A)\ \int \frac{x-12}{x^2-4x}dx$$

$$B)\ \int \frac{16-x}{x^2+3x-10}dx$$

	$C)\ \int \frac{2}{2x^2+3x+1}dx$ $D)\ \int \frac{x^3-5}{x^2-1}dx$

	$E)\ f'(x) = \dfrac{2x^3}{x^3 - x}$

	$F)\int\frac{9}{x^2+1}$ $G)\int\frac{9x}{x^2+1}$ $H)\int\frac{9x}{x^2-1}$ $I)\int\frac{9}{x^2-1}$

What you'll Learn About

- How to recognize a logistical growth differential equation

I am sick (Initial Value). Eventually everyone gets sick(Max). So what happens to the rate of people getting sick. People will get sick quickly, then it will be harder to find people that aren't sick yet (rate slows down-point of inflection) and eventually everyone gets sick.

This is similar to a rumor spreading or facebook/twitter accounts.

$$\frac{dP}{dt} = kP(M-P)$$

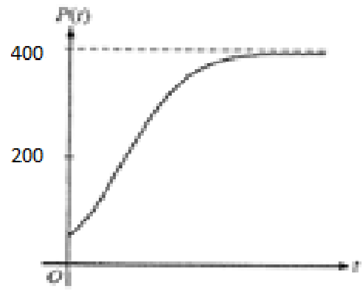
$$\frac{dP}{dt} \text{ rate of growth of people getting sick}$$

$$\text{kp: directly proportional to the sick people}$$

$$\text{M - P: healthy people (Not sick yet)}$$

Remember directly proportional is just like P = 8.50h (Your pay is directly proportional to the amount of money you make which can change) That 8.50 is your k.

	In 1985 and 1987, the Michigan Department of Natural Resources airlifted 61 moose form Algonquin Park, Ontario to Marquette County in the Upper Peninsula. It was originally hoped that the population P would reach carrying capacity in about 25 years with a growth rate of $\frac{dp}{dt} = .0003P(1000 - P)$ Solve the differential equation with the initial condition P(0) = 61.



24. Which of the following differential equations for a population P could model the logistic growth shown in the figure above?

A) $\frac{dP}{dt} = .02P - 0.0008P^2$ B) $\frac{dP}{dt} = .08P - .0002P^2$

C) $\frac{dP}{dt} = .8P^2 - 0.0002$ D) $\frac{dP}{dt} = 0.08P^2 - .0002$

E) $\frac{dP}{dt} = 0.08P^2 - 0.0002P$

21. The number of moose in a national park is modeled by the function M that satisfies the logistic differential equation $\frac{dM}{dt} = .05M\left(1 - \frac{M}{1000}\right)$, where t is the time in years and M(0)=50. What is the $\lim_{t\to\infty} M(t)$?

A) 50 B) 200 C) 500 D) 1000 E) 2000

84. The rate of change, $\frac{dP}{dt}$, of the number of people on an ocean beach is modeled by a logistic differential equation. The maximum number of people allowed on the beach is 1000. At 10 A.M., the number of people on the beach is 400 and is increasing at the rate of 200 people per hour. Which of the following differential equations describes the situation.

A) $\frac{dP}{dt} = \frac{1}{200}(1000 - P)$

B) $\frac{dP}{dt} = \frac{1}{2}P(1000 - P) + 100$

C) $\frac{dP}{dt} = \frac{1}{3}(1000 - P)$

D) $\frac{dP}{dt} = \frac{1}{1200}P(1000 - P)$

E) $\frac{dP}{dt} = 200P(1000 - P)$

26. The population P(t) of a species satisfies the logistic differential equation $\frac{dP}{dt} = P\left(4 - \frac{P}{2000}\right)$, where the initial position P(0)=1500 and t is the time in years. What is $\lim_{t\to\infty} P(t)$?

A) 2500 B) 8000 C) 4200 D) 2000 E) 4000

Let g be a function with g(4) = 1, such that all points (x, y) on the graph of g satisfy the logistic differential equation $\frac{dy}{dx} = 3y(2-y)$.

b) Given that g(3) = 1, find $\lim_{x\to\infty} g(x)$ and $\lim_{x\to\infty} g'(x)$.

c) For what value of y does the graph of g have a point of inflection? Find the slope of the graph of g at the point of inflection. (It is not necessary to solve for g(x).)

	A population is modeled by a function P that satisfies the logistic differential equation $\frac{dP}{dt} = \frac{P}{10}\left(1 - \frac{P}{15}\right)$. a) If P(0) = 3, what is the $\lim_{t\to\infty} P(t)$? If P(0) = 20, what is the $\lim_{t\to\infty} P(t)$? b) If P(0) = 3, for what value of P is the population growing the fastest? Rogawski 8. If y(x) is a solution to $\frac{dy}{dx} = 3y(10-y)$ with y(0) = 3 then as $x \to \infty$, A) y(x) increases to ∞ B) y(x) increases to 5 C) y(x) decreases to 5 D) y(x) increases to 10 E) y(x) decreases to 10 9. If y(x) is a solution to $\frac{dy}{dx} = 4y(12-y)$ with y(0) = 10 then as $x \to \infty$, A) y(x) decreases to ∞ B) y(x) increases to 6 C) y(x) increases to 12 D) y(x) decreases to 12 E) y(x) decreases to 0 16. If $\frac{dy}{dt} = 3y(10-2y)$ with y(0) = 1 then, y is increasing the fastest when A) y = 1.5 B) y = 2.5 C) y = 3 D) y = 4 E) y = 5 18. If $\frac{dy}{dt} = 3y(10-2y)$ with y(0) = 1, then the maximum value of y is A) y = 1 B) y = 2.5 C) y = 5 D) y = 10 E) Never attained; has no maximum value

Princeton Review (p. 806)

25. Given the differential equation $\frac{dz}{dt} = z\left(6 - \frac{z}{50}\right)$, where z(0) = 50, what is the $\lim_{t \to \infty} z(t)$?

A) 50 B) 100 C) 300 D) 6 E) 200

25. Given the differential equation $\frac{dz}{dt} = z\left(6 - \frac{z}{50}\right)$, where z(0) = 50, then z is increasing the fastest when z =

A) 150 B) 100 C) 300 D) 50 E) 100

Other Rate type problems

Rogawski

11. The rate at which a certain disease spreads is proportional to the quotient of the percentage of the population with the disease and the percentage of the population that does not have the disease. If the constant of proportionality is .03, and y is the percent of people with the disease, then which of the following equations gives R(t), the rate at which the disease is spreading.

$A)$ R(t) = .03y

B) R(t) = $\frac{.03dy}{dt}$

$C)$ $\frac{\text{dr}}{\text{dt}} = \frac{.03R}{(1-R)}$

D) R(t) = $.03\frac{\text{y}}{\text{(1 - y)}}$

$E)$ $\frac{\text{dr}}{\text{dt}} = .03R$

	12. The rate of change of the volume, V, of water in a tank with respect to time, t is directly proportional to the square root of the time, t, it takes to empty the tank . Which of the following is a differential equation that describes this relationship. $A)\ V(t) = k\sqrt{t}$ **B)** $V(t) = k\sqrt{V}$ $C)\ \frac{\text{dV}}{\text{dt}} = k\sqrt{t}$ $D)\ \frac{\text{dV}}{\text{dt}} = \frac{k}{\sqrt{V}}$ $E)\ \frac{\text{dV}}{\text{dt}} = k\sqrt{V}$ 16. Let P(t) represent the number of wolves in a population at time t years, when $t \geq 0$. The population P(t) is increasing at rate directly proportional to 500 divided by P(t), where the constant of proportionality is k. Write the differential equation that describes this relationship. 23. If P(t) is the size of a population at time t, which of the following differential equations describes exponential growth in the size of the population. $A)\ \frac{\text{dP}}{dt} = 200$ $B)\ \frac{\text{dP}}{dt} = 200t$ $C)\ \frac{\text{dP}}{dt} = 100t^2$ $D)\ \frac{\text{dP}}{dt} = 200P$ $E)\ \frac{\text{dP}}{dt} = 100P^2$

What you'll Learn About

- How to integrate functions that approach infinity or functions that approach an asymptote

2) $\int_1^\infty \frac{dx}{x^{1/3}}$

6) $\int_1^\infty \frac{2dx}{x^3}$

	10) $\int_{-\infty}^{0} \frac{dx}{(x-2)^3}$ 14) $\int_{-\infty}^{0} \frac{2dx}{x^2-4x+3}$

	18) $\int_{-\infty}^{0} x^2 e^x dx$ 43. Find the area of the region in the first quadrant that lies under the given curve $y = \frac{\ln x}{x^2}$

	22) $\int_{-\infty}^{\infty} 2xe^{-x^2}\,dx$ 26) $\int_0^1 \frac{dx}{\sqrt{1-x^2}}$

	30) $\int_{-1}^{4} \frac{dx}{\sqrt{\lvert x \rvert}}$ 41) $\int_{0}^{2} \frac{dx}{1-x}$