

CALCULUS: Graphical, Numerical, Algebraic by Finney, Demana, Waits and Kennedy
Chapter 3: Derivatives 3.9: Derivatives of Exponential and Logarithmic Functions

What you'll Learn About

How to take the derivative of exponential and logarithmic functions

A) $y = 5^x$

$$y = 5^x$$

$$y' = 5^x \ln(5) \cdot 1$$

(original) (ln base) $\frac{d}{dx}$ (power)

C) $y = 5^{\sin x}$

$$y = 5^{\sin x}$$

$$y' = 5^{\sin x} \cdot \ln 5 \cdot \cos x$$

E) $y = e^x$

$$y = e^x$$

$$y' = e^x \cdot \ln e \cdot 1 (=e^x)$$

G) $y = (5e)^{5x}$

$$y = (5e)^{5x}$$

$$y' = (5e)^{5x} \cdot \ln(5e) \cdot 5$$

I) $y = x^3 e^{4x} - x^4 e^{2x}$

$$y = (x^3 e^{4x}) - (x^4 e^{2x})$$

$$y' = x^3 (e^{4x} \cdot \ln e \cdot 4) + e^{4x} (3x^2) - \left[x^4 (e^{2x} \ln e \cdot 2) + e^{2x} \cdot 4x^3 \right]$$

B) $y = 7^{x^2}$

$$y = 7^{x^2}$$

$$y' = 7^{x^2} \cdot \ln 7 \cdot 2x$$

D) $y = 6^{\arctan x^3}$

$$y = 6^{\arctan(x^3)}$$

$$y' = 6^{\arctan(x^3)} \cdot \ln 6 \cdot \left(\frac{1}{1+(x^3)^2} \cdot 3x^2 \right)$$

F) $y = 5e^{5x}$

$$y = 5(e^{5x})$$

$$y' = 5 e^{5x} \ln e \cdot 5 = 25e^x$$

H) $y = e^{-\frac{3}{4}x}$

$$y = e^{-\frac{3}{4}x}$$

$$y' = e^{-\frac{3}{4}x} \cdot \ln e \cdot -\frac{3}{4}$$

B) ~~$y = 7^{x^2}$~~

$$y' = \frac{-3}{4} e^{-\frac{3}{4}x}$$

$$\frac{1}{3x \ln b}$$

$$A) y = \log_5(x^3)$$

$$y = \log_5(x^3)$$

$$y' = \frac{3x^2}{(x^3) \ln 5} = \frac{3}{x \ln 5}$$

$$B) y = \log_6 \sqrt[3]{x}$$

$$y = \log_6(x^{1/3})$$

$$y' = \frac{1}{x^{1/3} \cdot \ln 6} \cdot \frac{1}{3} x^{-2/3} = \frac{1}{3x^{2/3} x^{1/3} \ln 6}$$

$$C) y = \log_5\left(\frac{4}{x}\right)$$

$$y = \log_5(4x^{-1})$$

$$y' = \frac{1}{(4x^{-1}) \ln 5} \cdot -4x^{-2}$$

$$D) y = \frac{5}{\log_7(x^2)}$$

$$y = 5(\log_7(x^2))^{-1}$$

$$y' = -5(\log_7(x^2))^{-2} \cdot \frac{1}{x^2 \ln 7} \cdot 2x$$

$$E) y = \ln x$$

$$y = \ln x$$

$$\frac{dy}{dx} = \frac{1}{x \cdot \ln e} = \frac{1}{x}$$

$$F) y = \ln(x^4)$$

$$y = \ln(x^4)$$

$$y' = \frac{1}{x^4 \cdot \ln e} \cdot 4x^3 = \frac{4}{x}$$

$$y = \ln^4 x \quad \leftarrow$$

$$G) y = (\ln x)^4$$

$$y = (\ln x)^4$$

$$y' = 4(\ln x)^3 \cdot \frac{1}{x}$$

$$H) y = \ln\left(\frac{5}{x}\right) = \ln(5x^{-1})$$

$$y' = \frac{1}{(5x^{-1}) \ln e} \cdot -5x^{-2} = \frac{-x}{x^2} = \frac{-1}{x}$$

$$I) y = x^3 \ln(x^2) - \ln(\ln(\arcsinx))$$

$$y = (x^3 \ln(x^2)) - \ln(\underline{\ln(\arcsinx)})$$

$$y' = \left[x^3 \left(\frac{1}{x^2 \ln e} \cdot 2x \right) + \ln(x^2) \cdot 3x^2 \right] -$$

$$2x^2 + 3x^2 \ln(x^2) - \left(\frac{1}{\ln \arcsinx \cdot \ln e} \cdot \frac{1}{\arcsinx \ln e} \cdot \frac{1}{\sqrt{1-x^2}} \right)$$