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Intermediate Value Theorem

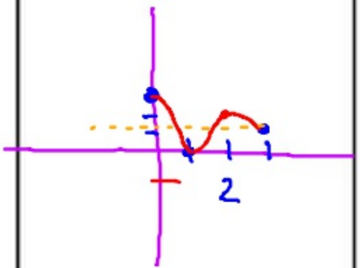
t (minutes)	0	2	5	8	12
$v_A(t)$ (meters/min)	0	100	40	-120	-150

1(FR). Train A runs back and forth on an east-west section of railroad track. Train A's velocity, measured in meters per minute, is given by a differentiable continuous function $v_A(t)$, where time t is measured in minutes. Selected values for $v_A(t)$ are given in the table above.

b) Do the data in the table support the conclusion that train A's velocity is 60 meters per minute at some time t with $2 < t < 5$? Give a reason for your answer.

yes, since the function is differentiable between $v(2)=100$ and $v(5)=40$ the train has to reach 60 m/min.

x	0	1	2	3
f(x)	3	0	k	1



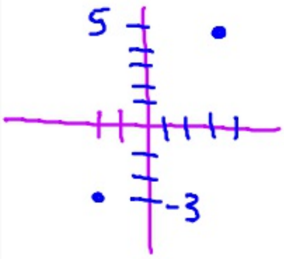
2(MC). The function f is continuous on the closed interval [0, 3] and has values that are given in the table above. The equation $f(x) = 1.5$ must have at least 3 solutions in the [0, 3] if k =

The graph must cross y = 1.5 three times

A) -1 B) 0 C) .5 D) 1 E) 2

3(MC). Let f be a continuous function on the closed interval [-2, 4]. If $f(-2) = -3$ and $f(4) = 5$, then the Intermediate Value Theorem guarantees that

(4,5) (-2,-3)



A) $f(c) = 1$ for at least one c between -3 and 5

B) $-3 \le f(x) \le 5$ for all x between -2 and 4

C) $f'(c) = \frac{4}{3}$ for at least one value of c between -2 and 4

D) $f(1) = 2$

E) $f(c) = 2$ for at least one c between -2 and 4

$h(4) = f(g(4)) - 6$	The functions f and g are differentiable for all real numbers, and g is strictly increasing. The table gives values of the functins and their first derivatives (their slopes) at selected values of x. the function h is given by h(x) = f(g(x)) - 6

x	f(x)	$f'(x)$	g(x)	$g'(x)$
1	6	3	2	1
2	9	1	$\boxed{3}$	1
3	10	4	4	2
4	11	1	6	6
5	12	1	12	6
6	13	2	18	7

4(FR) Explain why there must be a value r for 2 < r < 4 such that h(r) = 5.

$h(x) = f(g(x)) - 6$

$h(2) = f(g(2)) - 6$

$= f(3) - 6$

$= 10 - 6$

$h(2) = 4$

If x is between 2 and 4 there will be a y value of 5.

t (hours)	0	1	3	4	7	8	9
$L(t)$ (people)	120	156	176	126	150	80	0

5(FR). Concert tickets went on sale at noon (t = 0) and were sold out within 9 hours. The number of people waiting in line to purchase tickets at time t is modeled by the differentiable function L for $0 \le t \le 9$. Values of L(t) at various times t are shown in the table above.

How many times during the last 5 hours will L(t) equal 130? Give a reason for your answer.

Twice. Since L(t) is differentiable between L(4) = 126 and L(7) = 150 the number of people will equal 130.

What you'll Learn About

- How the derivative might fail to exist
- Differentiability implies local linearity
- Differentiability implies Continuity

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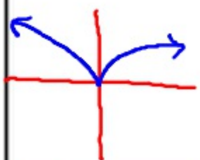
Differentiable

- Function is continuous and there is a slope at the specific point

Not a differentiable but continuous at

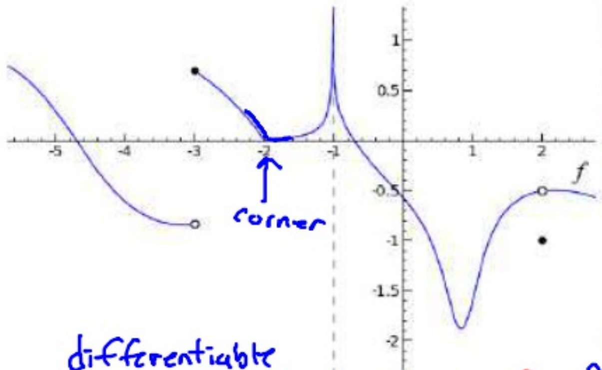
- corners
- vertical tangent
- cusp



Cusp: where 2 curves meet

Corner: where 2 lines or one line//curve meet

a. Find all points where the function, f(x), is differentiable.
b. Find all points where the function is continuous, but not differentiable.
c. Find all points where the graph is neither continuous nor differentiable.



corner

c) Not Differentiable
Not Continuous

$x = -3, -1, 2$

Jump VA Removable

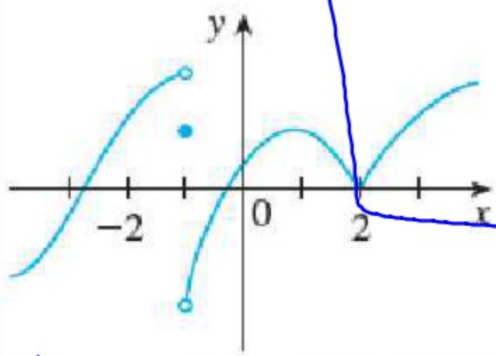
differentiable

corner

a) everywhere but at $x = -3, -2, -1, 2$

b) $x = -2$

c)



Not Differentiable (No slope)
Discontinuities

$x = -1$

~~Corner~~ cusp $x = 2$

a) everywhere but at $x = -1$ and $x = 2$

b) ~~corner~~ $(x = 2)$ cusp

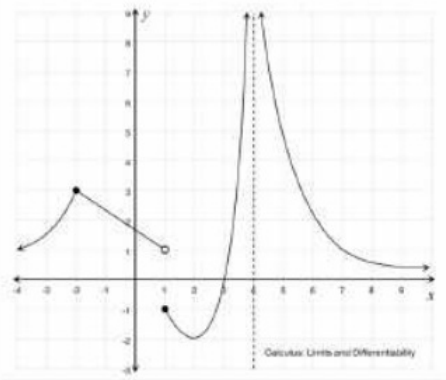
c) jump $(x = -1)$

3.2 Differentiability:

a. Find all points where the function, f(x), is differentiable.

corner|cusps|V.T ← b. Find all points where the function is continuous, but not differentiable.

VA|Holes|Jumps ← c. Find all points where the graph is neither continuous nor differentiable.



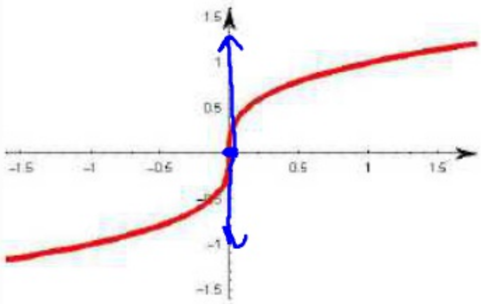
a) everywhere but

$x = -2, 1, 4$

corner jump VA

b) $x = -2$ (corner)

c) ~~x=-2~~, 1, 4



not differentiable at $x = 0$

Vertical tangent

No Slope ←	Not Differentiable at Removeable Discontinuities Vertical Asymptotes Jumps Corners Cusps Vertical Tangents