

CALCULUS: Graphical, Numerical, Algebraic by Finney, Demana, Waits and Kennedy
Chapter 2: Limits and Continuity 2.2: Limits Involving Infinity pg. 70-77

What you'll Learn About

- Finite Limits as x approaches positive or negative infinity
- End Behavior Models
- Infinite Limits as x approaches a value

Find the limit as $x \rightarrow \pm\infty$

$$4A) \lim_{x \rightarrow \infty} \frac{x+3}{3x^3 - x + 1} =$$

$$4B) \lim_{x \rightarrow \infty} \frac{3x^3 - x + 1}{x + 3} =$$

$$4C) \lim_{x \rightarrow \infty} \frac{3x^3 - x + 1}{x^3 + 3} =$$

$$4D) \lim_{x \rightarrow \infty} \frac{5x^2 - x + 2}{5x^2 + 10}$$

$$4E) \lim_{x \rightarrow -\infty} \frac{x^3 + x - 1}{x^2 - 5x + 2}$$

$$22A) \lim_{x \rightarrow \infty} \left(\frac{5}{x} + 2 \right) \left(\frac{8x^2 - 1}{2x^2} \right)$$

$$22B) \lim_{x \rightarrow \infty} \left(\frac{5}{x} + 2 \right) \left(\frac{8x^2 - 1}{2x^2} \right)$$

$$22C) \lim_{x \rightarrow \infty} \left(\frac{5}{x} + 2 \right) \left(\frac{8x^3 - 1}{2x^2} \right)$$

$$22D) \lim_{x \rightarrow \infty} \left(\frac{5}{x} + 2 \right) \left(\frac{8x^3 - 1}{2x^2} \right)$$

$$\lim_{x \rightarrow -2^+} \frac{x}{x+2} = -\infty$$

Find the limit as x approaches a number

$$14A) \lim_{x \rightarrow -2^+} \frac{x}{x+2} = -\frac{2}{0}$$

$$x = -1.9 \quad \text{Vertical asy}$$

$$y = \frac{-1.9}{-1.9+2} = \frac{-1.9}{.1} = -19$$

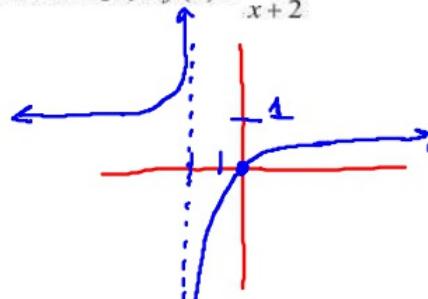
$$14B) \lim_{x \rightarrow -2^-} \frac{x}{x+2} = \infty$$

$$x = -2.1$$

$$y = \frac{-2.1}{-2.1+2} = \frac{-2.1}{-.1} = 21$$

$$14C) \lim_{x \rightarrow -2} \frac{x}{x+2} = \text{DNE}$$

Using the limits found above graph $f(x) = \frac{x}{x+2}$



$$\lim_{x \rightarrow \pm\infty} f(x) = 1$$

Describe

$$\lim_{x \rightarrow -\frac{1}{2}^-} f(x) = \infty$$

$$x = -0.6 \quad y = \frac{+}{(-\infty)}$$

$$\lim_{x \rightarrow -\frac{1}{2}^+} f(x) = -\infty$$

$$x = -0.4 \quad y = \frac{+}{(+\infty)} = -\infty$$

$$\lim_{x \rightarrow -\frac{1}{2}} f(x) = \text{DNE}$$

$$\lim_{x \rightarrow \infty} f(x) = 0$$

$$\lim_{x \rightarrow -\infty} f(x) = 0$$

Find the vertical asymptotes of the graph and then describe the behavior to the left and right of the vertical asymptote

$$30. f(x) = \frac{1-x}{2x^2-5x-3}$$

$$\frac{1-x}{(2x+1)(x-3)}$$

Describe

$$\lim_{x \rightarrow 3^-} f(x) = +\infty$$

$$x = 2.9 \quad y = \frac{+}{(-\infty)}$$

$$\lim_{x \rightarrow 3^+} f(x) = -\infty$$

$$x = 3.1 \quad y = \frac{+}{(+\infty)}$$

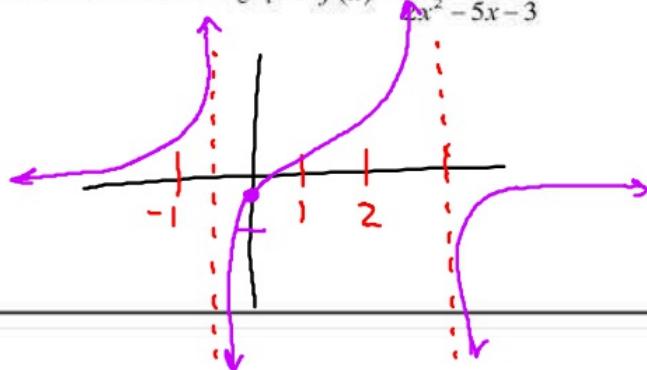
$$\lim_{x \rightarrow 3} f(x) = \text{DNE}$$

VA:

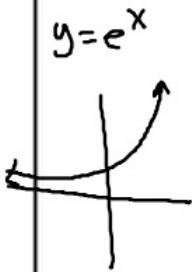
$$2x+1 = 0 \quad x = -\frac{1}{2}$$

$$x-3 = 0 \quad x = 3$$

Use your results from above to sketch a graph of $f(x) = \frac{1-x}{2x^2-5x-3}$



$$e^{-x} = \frac{1}{e^x}$$



Find the limit of the functions that involve e^x

3. $\lim_{x \rightarrow \infty} \frac{e^{-x}}{x} = 0$

$$\lim_{x \rightarrow -\infty} \frac{e^{-x}}{x} = \infty$$

A) $\lim_{x \rightarrow \infty} \frac{e^x + 2x}{2x} =$

$$\lim_{x \rightarrow \infty} \frac{e^x}{2x} + \frac{2x}{2x}$$

$$\lim_{x \rightarrow \infty} \left(\frac{e^x}{2x} + 1 \right) = \infty$$

B) $\lim_{x \rightarrow -\infty} \frac{e^x + 2x}{2x} =$

$$\lim_{x \rightarrow -\infty} \frac{e^x}{2x} + 1 =$$

$$0 + 1 = 1$$

Find the limit of the functions that involve sine and cosine

C) $\lim_{x \rightarrow -\infty} \frac{x^3 + \cos x}{x^3}$

$$\lim_{x \rightarrow -\infty} \frac{x^3}{x^3} + \frac{\cos x}{x^3}$$

$$\lim_{x \rightarrow -\infty} 1 + \frac{\cos x}{x^3} = 1$$

D) $\lim_{x \rightarrow +\infty} \frac{x^3 + \cos x}{x^3}$

$$\lim_{x \rightarrow \infty} 1 + \frac{\cos x}{x^3} = 1$$

E) $\lim_{x \rightarrow \infty} \sin\left(\frac{1}{x}\right)$

$$\sin 0 = 0$$

F) $\lim_{x \rightarrow \infty} \frac{\sin\left(\frac{1}{x}\right)}{1 + \frac{1}{x}} = \frac{0}{1 + 0} = 0$

Find the limit of the functions that involve absolute value

8A) $\lim_{x \rightarrow \infty} \frac{5x - 2}{|x| - 1} = 5$

8B) $\lim_{x \rightarrow -\infty} \frac{5x - 2}{|x| - 1} = -5$