	10. Consider the curve defined by the equation $x^2 + xy + y^2 = 27$ a) Write an expression for the slope of the curve at any point (x, y). $2x + x\frac{dy}{dx} + y + 2y\frac{dy}{dx} = 0$ $x\frac{dy}{dx} + 2y\frac{dy}{dx} = -2x - y$ $\frac{dy}{dx}(x+2y) = -2x-y$ $\frac{dy}{dx} = \frac{-2x-y}{x+2y}$
$(-6, 3)$ $(6, -3)$	b) Find the points on the curve where the lines tangent to the curve are vertical. (slope is undefined) $x+2y=0$ $x=-2y$ $x^2+xy+y^2=27$ $(-2y)^2+(-2y)(y)+y^2=27$ $4y^2-2y^2+y^2=27$ $3y^2=27$ $y^2=9$ $y=\pm 3$
$\frac{dy}{dx} = \frac{-2x-y}{x+2y}$	c) Find $\frac{d^2y}{dx^2}$ in terms of ~~x~~ and y $\frac{d^2y}{dx^2} = \frac{(x+2y)\left(-2-\frac{dy}{dx}\right) - (-2x-y)\left(1+2\frac{dy}{dx}\right)}{(x+2y)^2}$ $= \frac{(x+2y)\left(-2-\left(\frac{-2x-y}{x+2y}\right)\right) - (-2x-y)\left(1+2\left(\frac{-2x-y}{x+2y}\right)\right)}{(x+2y)^2}$

Consider the curve defined by the equation $2y^3 + 6x^2y - 12x^2 + 6y = 1$ with $\dfrac{dy}{dx} = \dfrac{4x - 2xy}{x^2 + y^2 + 1}$

b) Write an equation of each horizontal tangent to the curve

c) The line through the origin with slope -1 is tangent to the curve at point P. Find the x and y-coordinates of P.

d) Find $\dfrac{d^2y}{dx^2}$ in terms of y.

4.2/4.5: Linearization/Differentials/Mean Value Theorem pg. 196-204

What you'll Learn About
Linearization is another term for tangent line
Differentials are part of the derivative
Mean Value Theorem

a) Find the linearization of the function. b) Find L(a + .1) and f(a + .1)
c) Using concavity, determine if the Tangent Line at a is an overestimate or an underestimate. Justify your answer.

2. $f(x) = x^2 - 2x + 3 \quad a = 2$

1. $f(x) = \sqrt{1 + x} \quad a = 0$

	Find dy and evaluate dy for the given value of x and dx 20) $y = \frac{2x}{1+x^2}$ $x = -2$ and $dx = .1$ 24) $y = 3\csc\left(1 - \frac{x}{3}\right)$ $x = 1$ and $dx = .1$

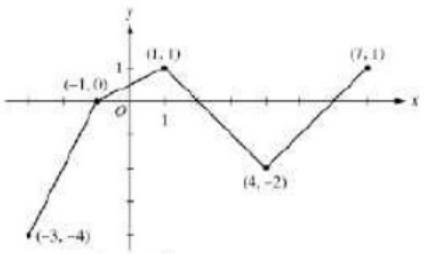
	Use the Mean Value Theorem to determine where the slope of the secant line equals the slope of the tangent line A) $f(x) = x^2$ [2,4] *B*) $f(x)= x^{\frac{1}{3}}$ [1,8] *C*) $f(x)= x^{\frac{1}{3}}$ [0,1] D) $f(x) = x^2$ [-2,2] 2013 BC3

Hot water is dripping through a coffeemaker, filling a large cup with coffee. The amount of coffee in the cup at time t, $0 \leq t \leq 6$, is given by a differentiable function C, where t is measured in minutes. Selected values of C(t), measured in ounces, are given in the table.

t(minutes)	0	1	2	3	4	5	6
C(t) ounces	0	5.3	8.8	11.2	12.8	13.8	14.2

Is there a time t, $3 \leq t \leq 6$, at which $C'(t) = 1$. Justify your answer.

Let g be a continuous function with g(2) = 5. The graph of the piecewise-linear function
g', the derivative of g , is shown for $-3 \leq x \leq 7$.

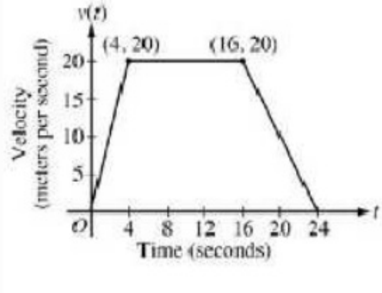


Graph of g'

Find the average rate of change of $g'(x)$, on the interval $-3 \leq x \leq 1$. Does the Mean Value Theorem applied on the interval $-3 \leq x \leq 1$ guarantee a value of c, for -3 < c < 1 , such that g'(c) is equal to this average rate of change? Why or why not?

2005 AB5

A car is traveling on a straight road. For $8 \le t \le 24$ seconds, the car's velocity v(t), in meters per second, is modeled by the piecewise-linear function defined by the graph



Find the average rate of change of v over the interval $0 \le t \le 16$. Does the Mean Value guarantee a value of c, for $0 < c < 16$, such that $v'(t)$ is equal to this average rate of change? Why of why not?

2004 BCB3

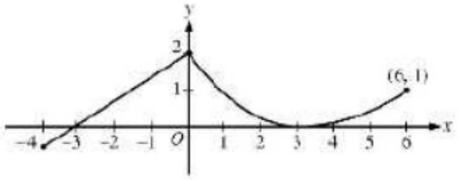
A test plane flies in a straight line with positive velocity v(t), in miles per minute at time t minutes, where v is a differentiable function of t. Selected values of v(t) are shown.

t(min)	0	5	10	15	20	25	30	35	40
v(t) (mpm)	7	9.2	9.5	9.2	4.5	2.4	4.5	4.9	7.3

Based on the values in the table, what is the smallest number of instances at which the acceleration of the plane could equal zero on the open interval $0 < t < 40$? Justify your answer

2009 BC3

A continuous function f is defined on the closed interval $-4 \le x \le 6$. The graph of f consists of a line segment and a curve that is tangent to the x-axis at x = 3, as shown in the figure above. On the interval 0 < x < 6, the function f is twice differentiable, with $f''(x) > 0$.



Graph of f

Is there a value a, for which the Mean Value Theorem, applied to the interval [a, 6], guarantees a value c, a < c < 6, at which $f'(c) = \frac{-1}{6}$? Justify your answer.

2011 BCB5

Ben rides a unicycle back and forth along a straight east-west track. The twice-differentiable function B models Ben's position of the track, measured in meters from the western end of the track, at time t, measured in seconds from the start of the ride. The table gives values of B(t) and Ben's velocity, v(t), measured in meters per second, at selected times t.

t(seconds)	0	15	40	60
B(t) (meters)	100	136	9	46
V(t) meters per second	2	2.3	2.5	4.6

For $15 \le t \le 60$, must there be a time t when Ben's velocity is -2 meters per second? Justify your answer.

92. Let f be the function defined by f(x) = x + ln(x). What is the value of c for

	which the instantaneous rate of change of f at x = c is the same as the average rate of change of f over [2, 6]? If $f(x) = \cos\left(\frac{x}{2}\right)$, then there exists a number c in the interval $\frac{\pi}{2} < x < \frac{3\pi}{2}$ that satisfies the conclusion of the Mean Value Theorem. Find those values.

(1) Find an equation

(2) Plug in only constant lengths

(3) Take derivative with respect to time

(4) Plug into derivative and solve

What you'll Learn About

How to use derivatives to solve a problem involving rates

A) ~~Water is draining~~ from a cylindrical tank with radius of 15 cm at 3000 cm^3/second. How fast is the surface dropping?

radius is constant

$V = \pi r^2 h$

$V = \pi (15^2) h$

$V = 225\pi h$

$\frac{dV}{dt} = 225\pi \frac{dh}{dt}$

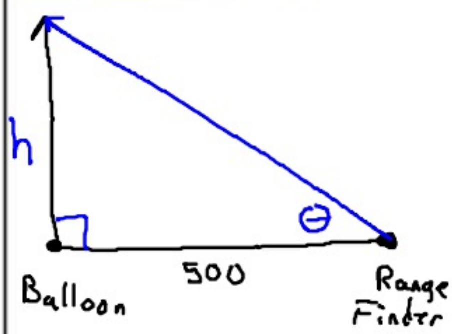
$\frac{dV}{dt} = 3000 \text{ cm}^3/\text{sec}$

Find $\frac{dh}{dt}$

$3000 = 225\pi \frac{dh}{dt}$

$\frac{3000}{(225\pi)}$ cm/sec $= \frac{dh}{dt}$

B) A hot-air balloon rising straight up from a level field is tracked by a range finder 500 ft from the lift-off point. At the moment the range finder's elevation angle is 45°, the angle is increasing at the rate of .14 rad/min.
How fast is the balloon rising at that moment?



$\boxed{\text{angle} = 45^\circ = \frac{\pi}{4} \quad \frac{d\theta}{dt} = .14 \frac{\text{rad}}{\text{min}} \quad \text{Find } \frac{dh}{dt}}$

$\tan = \frac{\text{opp}}{\text{adj}}$

$\frac{h}{500} = \frac{1}{500} h$

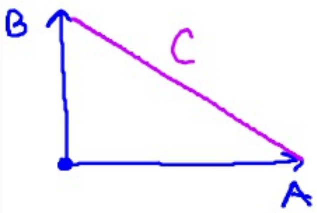
$\tan\theta = \frac{h}{500}$

$\sec^2\theta \cdot \frac{d\theta}{dt} = \frac{1}{500} \frac{dh}{dt}$

$(500)(\sec 45^\circ)^2(.14) = \frac{1}{500}\frac{dh}{dt}(500)$

$\frac{dh}{dt} = 500\left(\frac{2}{\sqrt{2}}\right)^2(.14)$

C) Truck A travels east at 40 mi/hr. Truck B travels north at 30 mi/hr. How fast is the distance between the trucks changing 6 minutes later?



$\frac{dA}{dt} = 40 \text{ mph}$ $\quad \frac{dB}{dt} = 30 \text{ mph}$

Find $\frac{dC}{dt}$ when $t = 6\text{min}$

$t = \frac{6}{60} = \frac{1}{10}\text{ hr}$

$$(A)^2 + B^2 = C^2$$

$$2A\frac{dA}{dt} + 2B\frac{dB}{dt} = 2C\frac{dC}{dt}$$

$$A(40) + B(30) = C\left(\frac{dC}{dt}\right)$$

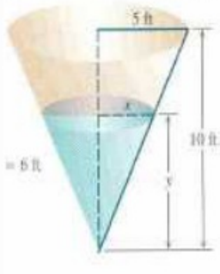
$$4(40) + 3(30) = 5\frac{dC}{dt}$$

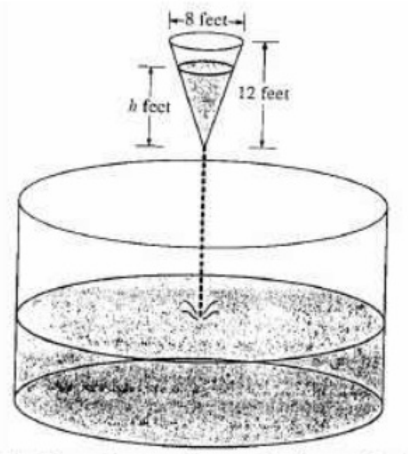
$$160 + 90 = 5\frac{dC}{dt}$$

$$250 = 5\frac{dC}{dt}$$

$$50\text{ mph} = \frac{dC}{dt}$$

D) Water runs into a conical tank at the rate of 9 ft^3/min.
The tank stands point down and has a height of 10 ft and a base radius of 5 ft. How fast is the water level rising when the water is 6 ft deep?



	21. Water is draining from a conical tank with height 12 feet and diameter 8 feet into a cylindrical tank that has a base with area 400π square feet. The depth, h, in feet, of the water in the conical tank is changing at the rate of (h - 12) feet per minute. Volume of a cone: $V = \frac{1}{3}\pi r^2 h$  A) Write an expression for the volume of water in the conical tank as a function of h. B) At what rate is the volume of water in the conical tank changing when h = 3? Indicate units of measure. C) Let y be the depth, in feet, of the water in the cylindrical tank. At what rate is y changing when h = 3? Indicate units of measure.

What you'll Learn About:
How to use derivatives to find limits in an indeterminate form

Why L'Hopitals Works

Sketch the graph of two curves with the following characteristic f(2) = g(2) = 0.

a) Write the tangent line for f(x)

b) Write the tangent line for g(x)

c) $\lim_{x \to 2} \frac{f(x)}{g(x)}$

d) $\lim_{x \to 0} \frac{2x^2}{x^2}$

2) $\lim_{x \to 0} \frac{\sin(5x)}{x}$

	4) $\lim_{x\to 1}\frac{\sqrt[3]{x}-1}{x-1}$ 49) $\lim_{x\to 1}\frac{x^3-1}{4x^3-x-3}$ A) $\lim_{x\to\infty}\frac{x^3-1}{4x^3-x-3}$ 27) $\lim_{x\to\infty}\frac{\ln(x^5)}{x}$ 35) $\lim_{x\to\infty}\frac{\log_2(x)}{\log_3(x+3)}$ 33) $\lim_{x\to 0}\frac{\sin(x^2)}{x}$