Differentiable → Instantaneous = Avg Rate

Hot water is dripping through a coffeemaker, filling a large cup with coffee. The amount of coffee in the cup at time t, $0 \le t \le 6$, is given by a differentiable function C, where t is measured in minutes. Selected values of C(t), measured in ounces, are given in the table.

t(minutes)	0	1	2	3	4	5	6
C(t) ounces	0	5.3	8.8	11.2	12.8	13.8	14.2

Is there a time t, $3 \le t \le 6$, at which $C'(t) = 1$. Justify your answer.

(3, 11.2) (6, 14.2)

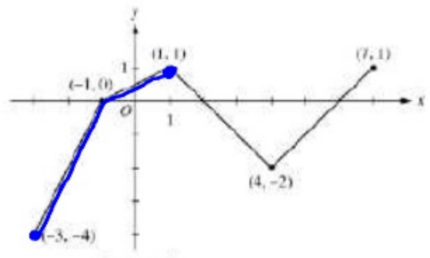
Instantaneous Rate

$$\frac{14.2 - 11.2}{6 - 3} = 1$$

Avg Rate = Instantaneous

yes. Because C(t) is differentiable and the Instantaneous rate of change = the avg. rate of change from $3 \le t \le 6$

Let g be a continuous function with g(2) = 5. The graph of the piecewise-linear function g', the derivative of g, is shown for $-3 \le x \le 7$.



Graph of g'

(-3, -4) (1, 1)

$$\text{Avg Rate} = \frac{1-(-4)}{1-(-3)} = \frac{5}{4}$$

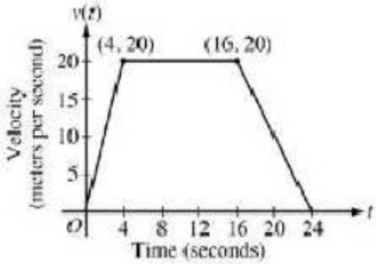
Find the average rate of change of $g'(x)$, on the interval $-3 \le x \le 1$. Does the Mean Value Theorem applied on the interval $-3 \le x \le 1$ guarantee a value of c, for -3 < c < 1, such that $g''(c)$ is equal to this average rate of change? Why or why not?

instantaneous

No. Because $g'(x)$ is not differentiable at $x = -1$

2005 AB5

A car is traveling on a straight road. For $8 \le t \le 24$ seconds, the car's velocity v(t), in meters per second, is modeled by the piecewise-linear function defined by the graph



$\text{Avg Rate} = \frac{20-0}{16-0} = \frac{20}{16}$

No, because $v(t)$ is not differentiable at $t=4$.

Find the average rate of change of v over the interval $0 \le t \le 16$. Does the Mean Value guarantee a value of c, for $0 < c < 16$, such that $v'(t)$ is equal to this average rate of change? Why of why not?

2004 BCB3

A test plane flies in a straight line with positive velocity v(t), in miles per minute at time t minutes, where v is a differentiable function of t. Selected values of v(t) are shown.

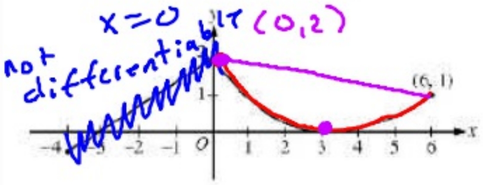
t(min)	0	5	10	15	20	25	30	35	40
v(t) (mpm)	7	9.2	9.5	9.2	4.5	2.4	4.5	4.9	7.3

Based on the values in the table, what is the smallest number of instances at which the acceleration of the plane could equal zero on the open interval $0 < t < 40$? Justify your answer

Two. Since $v(t)$ is differentiable the average rate of change = 0 when the values of $v(t)$ are the same. this happens between $5 \le t \le 15$ and $20 \le t \le 30$.

2009 BC3

A continuous function f is defined on the closed interval $-4 \le x \le 6$. The graph of f consists of a line segment and a curve that is tangent to the x-axis at x = 3, as shown in the figure above. On the interval 0 < x < 6, the function f is twice differentiable, with $f''(x) > 0$.



Graph of f

$(a,\ )(6,1)$
$(0,2)(6,1)$

C is just an x-value

Is there a value a, for which the Mean Value Theorem, applied to the interval [a, 6], guarantees a value c, a < c < 6, at which $f'(c) = \frac{-1}{6}$? Justify your answer.

instantantaneous

yes at a=0
since f is differentiable from $0 \le t \le 6$
and the avg rate of change = instantaneous rate of change

2011 BCB5

Ben rides a unicycle back and forth along a straight east-west track. The twice-differentiable function B models Ben's position of the track, measured in meters from the western end of the track, at time t, measured in seconds from the start of the ride. The table gives values of B(t) and Ben's velocity, v(t), measured in meters per second, at selected times t.

t(seconds)	0	15	40	60
B(t) (meters)	100	136	9	46
V(t) meters per second	2	2.3	2.5	4.6

For $15 \le t \le 60$, must there be a time t when Ben's velocity is -2 meters per second? Justify your answer.

$\text{Avg Rate} = \frac{136-46}{15-60}$ $\quad -2 =$ instantaneous

~~92. Let f be the function defined by f(x) = x + ln(x). What is the value of c for~~

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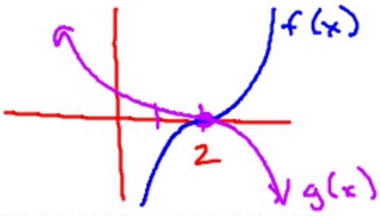
What you'll Learn About:
How to use derivatives to find limits in an indeterminate form

Why L'Hopitals Works

Sketch the graph of two curves with the following characteristic $\boxed{f(2) = g(2) = 0.}$



a) Write the tangent line for f(x)

$(2,0)$ $f'(x)$

$y = 0 + f'(x)(x-2)$

b) Write the tangent line for g(x)

$(2,0)$ $g'(x)$

$y = 0 + g'(x)(x-2)$

c) $\lim_{x \to 2} \frac{f(x)}{g(x)} = \frac{f(2)}{g(2)} = \frac{0}{0}$

$\lim_{x \to 2} \frac{f'(x)(x-2)}{g'(x)(x-2)} \to \boxed{\lim_{x \to 2} \frac{f'(x)}{g'(x)}}$

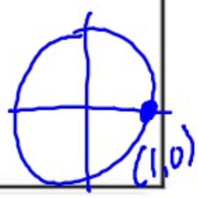
d) $\boxed{\lim_{x \to 0} \frac{2x^2}{x^2}} = \frac{0}{0}$

$\lim_{x \to 0} \frac{4x}{2x} = \frac{0}{0}$

$\lim_{x \to 0} \frac{4}{2} = 2$

2) $\lim_{x \to 0} \boxed{\frac{\sin(5x)}{x}} = \frac{0}{0}$

$\lim_{x \to 0} \frac{\cos(5x) \cdot 5}{1} = 5$



	4) $\lim_{x\to 1}\frac{\sqrt[3]{x}-1}{x-1} = \frac{0}{0}$ $\lim_{x\to 1}\frac{x^{1/3}-1}{x-1}$ $\lim_{x\to 1}\frac{\frac{1}{3}x^{-2/3}}{1} = \frac{1}{3}$ 49) $\lim_{x\to 1}\frac{x^3-1}{4x^3-x-3} = \frac{0}{0}$ $\lim_{x\to 1}\frac{3x^2}{12x^2-1} = \frac{3}{11}$ A) $\lim_{x\to\infty}\frac{x^3-1}{4x^3-x-3} = \frac{1}{4}$ Horizontal Asym: 0 27) $\lim_{x\to\infty}\frac{\ln(x^5)}{x} = \frac{\infty}{\infty}$ $\lim_{x\to\infty}\frac{1}{x^5 \ln e}\cdot 5x^4 = \frac{5x^4}{x^5} = \frac{5}{x}$
L.R. Rewrite LR	Indeterminate Form 35) $\lim_{x\to\infty}\frac{\log_2(x)}{\log_3(x+3)} = \frac{\infty}{\infty}$ $\lim_{x\to\infty}\frac{\left(\frac{1}{x\ln 2}\right)}{\left(\frac{1}{(x+3)\ln 3}\right)}$ $\lim_{x\to\infty}\frac{(x+3)\ln 3}{x\ln 2} = \frac{\ln 3}{\ln 2}$ $\lim_{x\to\infty}\frac{\ln 3}{\ln 2}$ 33) $\lim_{x\to 0}\frac{\sin(x^2)}{x} = \frac{0}{0}$ $\lim_{x\to 0}\frac{\cos(x^2)\cdot 2x}{1} = \frac{0}{1} = 0$