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AP CALCULUS
Stuff you **MUST** know Cold

* means topic only on BC

Curve sketching and analysis

$y = f(x)$ must be continuous at each:

critical point: $\frac{dy}{dx} = 0$ or undefined or endpoints

local minimum:

$\frac{dy}{dx}$ goes (–,0,+) or (–,und,+) or $\frac{d^2y}{dx^2} > 0$

local maximum:

$\frac{dy}{dx}$ goes (+,0,–) or (+,und,–) or $\frac{d^2y}{dx^2} < 0$

point of inflection: concavity changes

$\frac{d^2y}{dx^2}$ goes from (+,0,–), (–,0,+), (+,und,–), or (–,und,+)

Differentiation Rules

Chain Rule

$$\frac{d}{dx}\left[f(u)\right] = f'(u)\frac{du}{dx} \quad OR \quad \frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$$

Product Rule

$$\frac{d}{dx}(uv) = \frac{du}{dx}v + u\frac{dv}{dx} \quad OR \quad u'v + uv'$$

Quotient Rule

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{\frac{du}{dx}v - u\frac{dv}{dx}}{v^2} \quad OR \quad \frac{u'v - uv'}{v^2}$$

Approx. Methods for Integration

Trapezoidal Approximation

Right Riemann Sum Approximations

Left Riemann Sum Approximations

Midpoint Riemann Sum Approximations

Basic Derivatives

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$$

$$\frac{d}{dx}\left(\sin x\right) = \cos x$$

$$\frac{d}{dx}\left(\cos x\right) = -\sin x$$

$$\frac{d}{dx}\left(\tan x\right) = \sec^2 x$$

$$\frac{d}{dx}\left(\cot x\right) = -\csc^2 x$$

$$\frac{d}{dx}\left(\sec x\right) = \sec x \tan x$$

$$\frac{d}{dx}\left(\csc x\right) = -\csc x \cot x$$

$$\frac{d}{dx}\left(\ln u\right) = \frac{1}{u}\frac{du}{dx}$$

$$\frac{d}{dx}\left(e^u\right) = e^u\frac{du}{dx}$$

"PLUS A CONSTANT"

The Fundamental Theorem of Calculus

$$\int_a^b f(x)dx = F(b) - F(a)$$

where $F'(x) = f(x)$

AVERAGE VALUE

If the function $f(x)$ is continuous on $[a, b]$ and the first derivative exists on the interval (a, b), then there exists a number $x = c$ on (a, b) such that

$$f(c) = \frac{\int_a^b f(x)dx}{(b-a)}$$

This value $f(c)$ is the "average value" of the function on the interval $[a, b]$.

Corollary to FTC

$$\frac{d}{dx}\int_{a(x)}^{b(x)} f(t)dt = f(b(x))b'(x) - f(a(x))a'(x)$$

Intermediate Value Theorem

If the function $f(x)$ is continuous on $[a, b]$, and y is a number between $f(a)$ and $f(b)$, then there exists at least one number $x = c$ in the open interval (a, b) such that $f(c) = y$.

Solids of Revolution and friends

Disk Method

$$V = \pi\int_{x=a}^{x=b}\left[R(x)\right]^2 dx$$

Washer Method

$$V = \pi\int_a^b\left(\left[R(x)\right]^2 - \left[r(x)\right]^2\right)dx$$

General volume equation (not rotated)

$$V = \int_a^b Area(x)\, dx$$

*Arc Length $L = \int_a^b \sqrt{1 + \left[f'(x)\right]^2}\, dx = \int_a^b \sqrt{\left[x'(t)\right]^2 + \left[y'(t)\right]^2}\, dt$

More Derivatives

$$\frac{d}{dx}\left(\sin^{-1} u\right) = \frac{1}{\sqrt{1-u^2}}\frac{du}{dx}$$

$$\frac{d}{dx}\left(\cos^{-1} x\right) = \frac{-1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\left(\tan^{-1} x\right) = \frac{1}{1+x^2}$$

$$\frac{d}{dx}\left(\cot^{-1} x\right) = \frac{-1}{1+x^2}$$

$$\frac{d}{dx}\left(\sec^{-1} x\right) = \frac{1}{|x|\sqrt{x^2-1}}$$

$$\frac{d}{dx}\left(\csc^{-1} x\right) = \frac{-1}{|x|\sqrt{x^2-1}}$$

$$\frac{d}{dx}\left(a^x\right) = a^x \ln a$$

$$\frac{d}{dx}\left(\log_a x\right) = \frac{1}{x \ln a}$$

Mean Value Theorem

If the function $f(x)$ is continuous on $[a, b]$, AND the first derivative exists on the interval (a, b), then there is at least one number $x = c$ in (a, b) such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$

Somewhere the derivative equals the slope between the endpoints

Rolle's Theorem

If the function $f(x)$ is continuous on $[a, b]$, AND the first derivative exists on the interval (a, b), AND $f(a) = f(b)$, then there is at least one number $x = c$ in (a, b) such that

$$f'(c) = 0.$$

Distance, Velocity, and Acceleration

velocity = $\frac{d}{dx}$ (position)

acceleration = $\frac{d}{dx}$ (velocity)

*velocity vector = $\left\langle \frac{dx}{dt}, \frac{dy}{dt} \right\rangle$

speed = $|v| = \sqrt{(x')^2 + (y')^2}$ *

displacement = $\int_{t_o}^{t_f} v\, dt$

distance = $\int_{\text{initial time}}^{\text{final time}} |v|\, dt$

$\int_{t_o}^{t_f} \sqrt{(x')^2 + (y')^2}\, dt$ *

average velocity =

$$= \frac{\text{final position} - \text{initial position}}{\text{total time}}$$

$$= \frac{\Delta x}{\Delta t}$$

BC TOPICS and important TRIG identities and values

l'Hôpital's Rule

If $\frac{f(a)}{g(b)} = \frac{0}{0}$ or $= \frac{\infty}{\infty}$,

then $\lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a}\frac{f'(x)}{g'(x)}$

Slope of a Parametric equation

Given a $x(t)$ and a $y(t)$ the slope is

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

Values of Trigonometric Functions for Common Angles

θ	$\sin\theta$	$\cos\theta$	$\tan\theta$
0°	0	1	0
$\frac{\pi}{6}$,30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$
37°	3/5	4/5	3/4
$\frac{\pi}{4}$,45°	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
53°	4/5	3/5	4/3
$\frac{\pi}{3}$,60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
$\frac{\pi}{2}$,90°	1	0	"∞"
π,180°	0	–1	0

Euler's Method

If given that $\frac{dy}{dx}$ and that the solution passes through (x_o, y_o),

- Use a tangent line to build the curve

$$y = y_1 + \tfrac{dy}{dx}(x - x_1)$$

Polar Curve

For a polar curve $r(\theta)$, the **AREA** inside a "leaf" is

$$\int_{\theta_1}^{\theta_2} \tfrac{1}{2}\left[r(\theta)\right]^2 d\theta$$

where θ_1 and θ_2 are the "first" two times that $r = 0$.

The **SLOPE** of $r(\theta)$ at a given θ is

$$x = r\cos\theta \quad y = r\sin\theta$$

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$$

Tabular Integration – When one piece is not the derivative of the other

$$\int \ln x\,dx =$$

lnx	dx
1/x	x

$$\int \ln x\,dx = x\ln x - \int 1\,dx$$

$$\int \ln x\,dx = x\ln x - x + C$$

Ratio Test

The series $\sum_{k=0}^{\infty} a_k$ converges if

$$\lim_{k\to\infty}\left|\frac{a_{k+1}}{a_k}\right| < 1$$

If the limit equal 1, you know nothing.

Interval of convergence (Test endpoints)

L'hopitals Rule: When the limit of a function is $\frac{0}{0}$ or $\frac{\infty}{\infty}$

Take the derivative of the top and the derivative of the bottom and then re-evaluate the limit

Taylor Series

If the function f is "smooth" at $x = a$, then it can be approximated by the n^{th} degree polynomial

$$f(x) \approx f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \ldots + \frac{f^{(n)}(a)}{n!}(x-a)^n.$$

Take derivatives, plug in your center and divide by your factorials.

Lagrange Error Bound

If $P_n(x)$ is the n^{th} degree Taylor polynomial of $f(x)$ about c and $\left|f^{(n+1)}(t)\right| \le M$ for all t between x and c, then

$$\left|f(x) - P_n(x)\right| \le \frac{M}{(n+1)!}\left|x-c\right|^{n+1}$$

M=Maximum of the next derivative
(x-c) is the distance from center
(n+1)! Is the next derivative
$\left|f(x) - P_n(x)\right|$ is the actual error

Sum of an infinite geometric series

$$S = \frac{1^{st}\text{ term}}{1-r}$$

where r is the common ratio

Maclaurin Series

A Taylor Series about $x = 0$ is called Maclaurin.

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \ldots$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots$$

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \ldots$$

$$\ln(x+1) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots$$

Alternating Series Error Bound

If $S_N = \sum_{k=1}^{N}(-1)^n a_n$ is the N^{th} partial sum of a convergent alternating series, then

$$\left|S_\infty - S_N\right| \le \left|a_{N+1}\right|$$

This means error is less than the next term

Integration by Separation
Don't forget +C
Get y with dy and x with dx